Chapter 1
The Basics

Until recently, there was only one application for ray tracing, the modeling and analysis of an optical system. However, today's students have a different application in mind. They think that they are going to be taught how to use powerful computers to generate realistic scenes like those that animation studios use to create movies (see Fig. 1.1).

Both applications, optical system modeling and realistic scene generation, simulate rays traveling through space to create images. Some of these images may be very simple (such as a point or a line), while those in computer-generated images (CGI) are extremely complex.

In the latter case, thousands of rays are traced to build the image of a scene. To reduce the time to compute the scene, only those rays that will reach the eye are traced. The easiest way to do this is to trace the rays in reverse. Starting at an eye, or a camera sensor, a ray is traced from a point on the sensor, through the lens, and out to the scene, where its intersection with the surfaces defined by the computer model of the scene reflect, refract, and scatter the light in the scene.

The CGI procedure is designed to use as few rays as possible so that the images can be rapidly generated. Similarly, system analysis ray tracing tries to use the fewest rays possible to determine how well an optical system, such as that shown in Fig. 1.2, will perform if it were built. In this case, the scene doesn't change. Instead, the same rays are traced through many different variations of an optical system whose dimensions and other variables are changed to find the best performance under specific conditions. This operation is called optimization, and a substantial part of this text describes and demonstrates how it works.

Figure 1.1 “Bosque Morning,” by John Fowler. Reprinted with permission from Creative Commons.

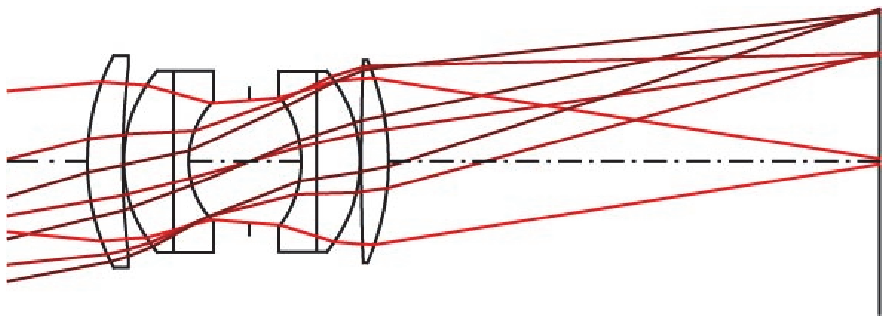


Figure 1.2 Ray trace of a double Gauss lens using an optical design program.

Although computers provide us with enormous power to solve complex and sophisticated problems, they represent our biggest concern in teaching optical design. Once you sit down in front of a computer, the rest of the world can disappear, and time can pass quickly. Useful ideas and concepts fade when you are typing and looking at numbers that are supposed to tell you how well things are going. The physics can get lost in front of a keyboard; basic insights can disappear inside the workings of the design program.

To resist this tendency, we will introduce certain basic concepts to help you visualize what is going on in a design or that can provide insight and guidance toward improving its performance. Some of these concepts are simple, but they are just as useful as the ability to run a series of advanced analyses on a system.

1.1 Ray Calculations

We use computers that perform complex calculations and, for the most part, do not wonder what the calculations are or which ones are being used. Until a ridiculous result or a bug presents itself, we're fine with the way things are going.

In the case of ray tracing, the equations are so simple and their approximations are sometimes so useful that it would be a shame to ignore them. First, we need to set up a coordinate system that will be used from here on out—even when we resort to computing. The coordinates are established by defining the z axis as the axis of symmetry of a lens (Fig. 1.3) or other optical component, such as a spherical mirror. Light from an object is directed into the optical system in a positive direction. Using the convention that light initially travels from left to right (see box, "The Ways of Rays"), the positive z axis points to the right. In the right-handed coordinate system used in optics, the x and y axes are perpendicular to the z axis. They are oriented with the positive y axis pointing up and the z axis to the right, and the x axis points away from the y-z plane. (If you curl the fingers of your right hand so that they point from the x axis to the y axis, your thumb will point along the z axis). Whew!

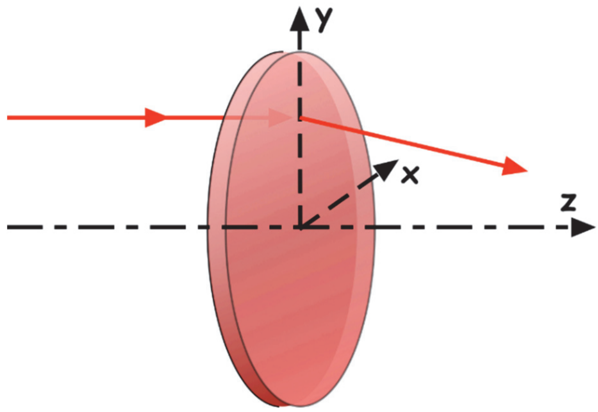


Figure 1.3 Right-handed coordinate system used in ray tracing.

The Ways of Rays

There are many mathematical and graphic conventions in optics and optical engineering, but one that is unstated, yet almost universal, is that light rays in an optical design or ray trace always travel from left to right, beginning at the object plane or source and entering the initial surface of the optical system. What happens after that is, of course, dependent on that particular system. Still, this convention means that you should expect to read an optical system as you would a sentence. Anyone who draws an optical system with the source on the right side of the design is, to our mind, guilty of a violation of optics grammar and should be told so.

The index of refraction n is a characteristic of a transparent medium (gas, liquid, or solid). When light travels through a vacuum, it travels at a velocity of 3×10^8 meters per second (m/s). It is denoted by a special symbol c. In any other medium, the interaction of the light with its atoms, molecules, or structures causes the light to slow down to a speed of v. To account for the velocity changes, each medium has a refractive index, which is the ratio of the speed of light divided by its speed in the medium:

$$n = c/v. \tag{1.1}$$

Even air, the medium between lenses and mirrors in most optical systems, slows the progress of light (it has an index of 1.000273 at standard room temperature and pressure). These indices are known as *absolute* indices. Because air (with an index very close to 1.0) is a medium present in nearly every lens design, it is standard practice in optical design software to redefine the materials in terms of their *relative* index, where the speed of light used in Eq. (1.1) is the speed of light in air (not vacuum), making the refractive index of air exactly equal to one and simplifying the material entry of the optical system. All standard glass catalogs also have indices expressed relative to this unit index of air.

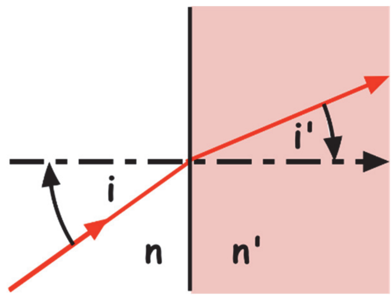


Figure 1.4 Refraction of light at an interface.

1.1.1 Law of refraction: Snell's law

The laws of ray optics can be expressed in three equations. The first law, known as Snell's law or the law of refraction, expresses the amount by which a light ray is bent when it crosses the interface between media of different refractive indices (Fig. 1.4). In the case of a plane surface, there is no unique optical axis as there is in the case of a lens. Yet, to be able to specify the quantities that describe the rays and the optics, a local coordinate system needs to be established. In this case, we use an incident ray to locate the origin. Starting at the interface that separates two media whose refractive indices are n and n', the origin of the coordinate system is located at the point where the incident ray intersects the interface. The z axis is defined as the normal (perpendicular) to the interface at the origin, as shown in Fig. 1.4. See the box, "Sign Conventions," for a summary of definitions for optical systems.

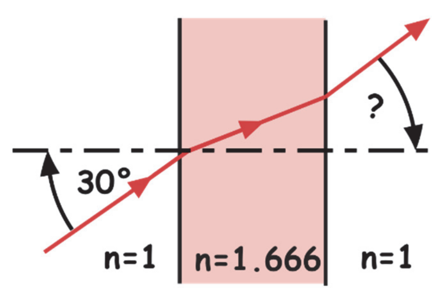
If the angle of the incident ray to the z axis is i, then the angle of refraction i' is given by the relation

$$n' \sin i' = n \sin i. \tag{1.2}$$

In Fig. 1.4, the signs of the angles i and i' are both positive. The sign convention is that the angle between a ray and a reference axis is positive if the rotation of the ray into the axis is clockwise. (For a horizontal reference axis, rays progressing upward are positive, and those going downward are negative.) Note that the equation is not solved for i' but expressed as sine-index products, which helps to emphasize the fact that this product is constant across index boundaries, and in the case of parallel plate of material, it can simplify some of the calculations (Ex. 1.1).

Exercise 1.1 Parallel slab

A ray enters a 1-cm-thick pane of glass in air (n = 1) at an angle of 30° to the surface normal. Its surfaces are parallel to each other. If the refractive index of the pane is 1.666, what is the exiting angle? The answers are given at the end of the chapter. (Hint: Examine the sine-index product at each interface.)



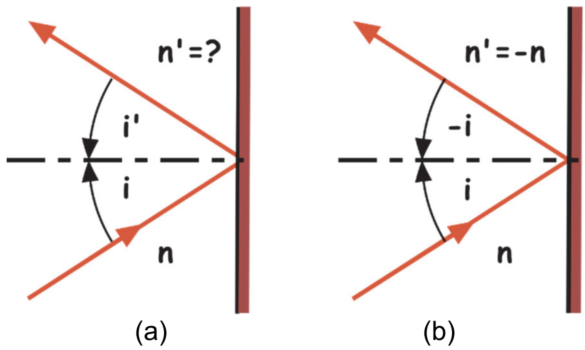


Figure 1.5 Reflection of light off a mirror.

1.1.2 Law of reflection

The second law, the law of reflection, states that the angle of reflection equals the angle of incidence. But how is this expressed algebraically? We can treat it as a special case of Snell's law, provided we establish another convention. One of the problems with ray tracing is that when a ray is reflected, its general direction must be reconciled with the coordinate system that we initially established with the z axis pointing in the direction of the light propagation. And it also has to be consistent with our angle convention (positive, if the ray rotates clockwise into the axis).

In Fig. 1.5(a), the refractive index in the space in front of the mirror n' should be the same for the reflected ray as it is for the incident ray. But the angles i and i' should have opposite signs ($i' = -i$). If we insert these values into the equation for Snell's law (Eq. 1.2), we obtain

$$n' \sin(-i) = n \sin i,$$

or

$$-n' \sin(i) = n \sin i.$$

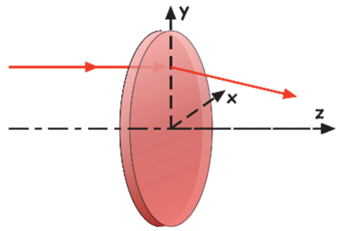
Therefore,

$$n' = -n.$$

The only time that this is true is when $i = 0$. It would appear that we couldn't use Snell's law universally and would have to set up a special case for reflections, which would make ray tracing more difficult because each time the ray encountered a reflecting surface, some additional branching would have to be added. One way that these computations can be completely consistent is if the refractive index changed sign after each reflection. So, $n' = -n$. When this is inserted into the equation for Snell's law, it is satisfied (Fig. 1.5(b)). In nature, the refractive index of optical materials is always positive (exceptions only occur in very exotic situations), but in ray tracing, the signed refractive index is used as a geometrical bookkeeping trick that reflects (pun intended!) the physics of light.

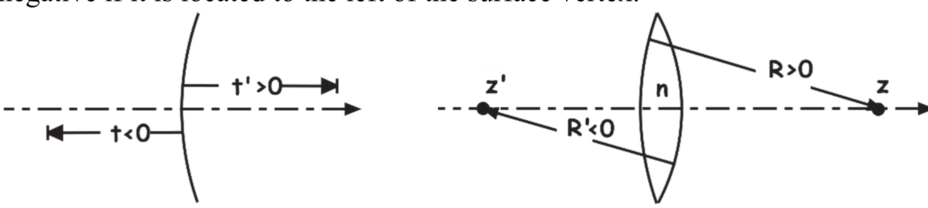
Sign Conventions

Optical axis
The optical axis, the *z* axis, is the axis of symmetry of the optical system. The positive direction points along the direction of the initial propagation of light (to the right).

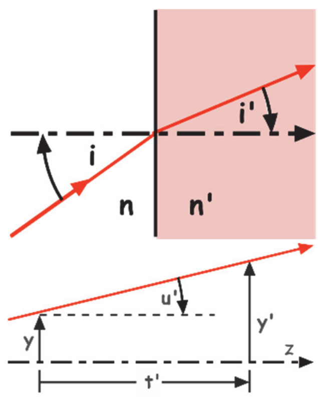


Coordinate origin
The coordinate system for ray tracing is right-handed. Its origin is located at the intersection of a surface and the optical axis. As a ray is propagated to the next surface, a new origin is established there.

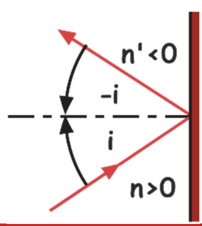
Distances
The distance to the next surface (thickness) is positive if it lies along the initial direction of light propagation (to the right) and negative if it is directed against the initial ray propagation (to the left). The radius of curvature of a surface is positive if the center of curvature is located to the right of the surface vertex and negative if it is located to the left of the surface vertex.



Rays
The height of a ray above the axis is positive; below the axis is negative. The angle of the ray with respect to a reference line is positive if the rotation of the ray into the reference is clockwise and negative if the rotation is counterclockwise. This is true for angles of incidence and refraction (i, i') measured relative to the surface normal and for ray angles (u, u') measured relative to the optical axis.



Reflections
All sign conventions remain the same except that the sign of the refractive index is changed after each reflection.



1.1.3 The transfer equation

The third and simplest of the laws is that in a medium of constant refractive index, light travels in a straight line. If a ray is y above the optical axis and at an angle u' to the optical axis (Fig. 1.6), after traveling a distance t' it will be a distance y' above the axis, where

$$y' = y + t' \tan u'. \tag{1.3}$$

Equation (1.3) is called the transfer equation, and in conjunction with Snell's law it is all we need to trace rays through a complex optical system.

Exercise 1.2 Transfer equation
A ray is traveling at a height of 10 mm above the optical axis with a slope angle of +0.2 radians. If it travels 50 mm farther along the optical axis, what will be its new height? When its height is 75 mm, how far will it have traveled from the point where it started at a height of 10 mm?

As an example of how is this done, let's look at a ray striking a spherical surface with a radius of curvature R, as shown in Fig. 1.7. We start with a ray at an angle u to the optical axis. It is incident on the spherical surface at a height y and an angle of incidence i with respect to the surface normal, which is an extension of the radius of the spherical surface. The ray angle after refraction i' can be calculated using Snell's law. Although it is possible to compute the direction of the ray after refraction using trigonometry, the analysis becomes much simpler for small ray

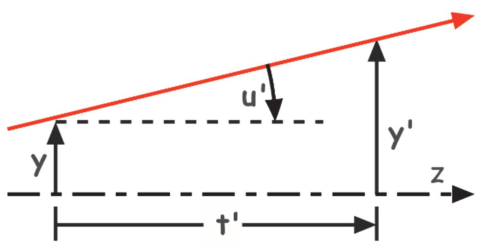


Figure 1.6 Transfer of a ray through a distance *t'*.

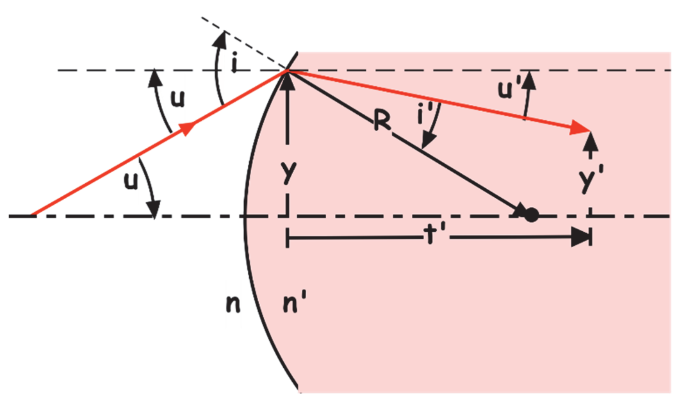


Figure 1.7 Ray trace at a spherical surface.

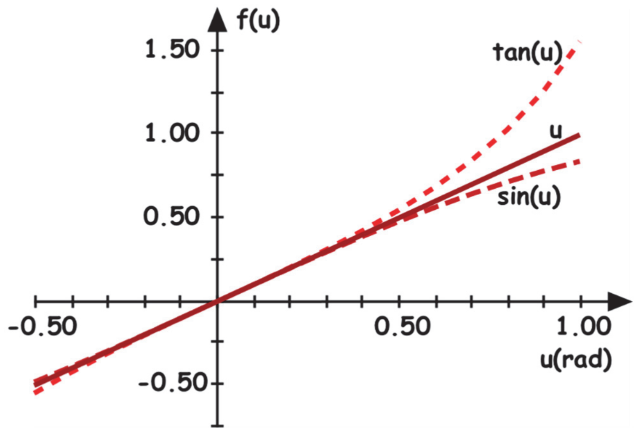


Figure 1.8 Demonstration of the small-angle approximation of trigonometric functions close to the origin.

angles in the paraxial region (close to the axis) where the sine and tangent of an angle are approximately equal to the angle in radians. As you can see in Fig. 1.8, the three functions (plotted as a function of the angle u) are coincident for a small range of angles about the origin, so that we can replace sin i with its angle i. Snell's law (Eq. (1.2)) is then reduced to

$$n'i' = ni, \tag{1.4}$$

and the transfer equation (Eq. (1.3)) is expressed as the paraxial transfer equation,

$$y' = y + t'u'. \tag{1.5}$$

To continue the ray trace, the angle that the ray makes with the optical axis after refraction u' must be determined. Because the refraction angles depend on where the ray hits the surface, Eq. (1.4) can be rewritten in terms of the ray angles with respect to the optical axis u and u' as

$$n'u' = nu - y\phi. \tag{1.6}$$

This is called the paraxial refraction equation. The term ϕ is the optical power of the surface and is related to the difference in the refractive indices across the surface and its radius of curvature R or its curvature c (the reciprocal of R) as

$$\phi = \frac{n' - n}{R} = (n' - n)c. \tag{1.7}$$

Note that for a flat surface ($R = \infty$) the optical power is zero and the paraxial refraction equation reduces to Snell's law because $u = i$ and $u' = i'$. Given the initial ray angle u, ray height y, and the power of the surface ϕ, one can calculate the ray angle on the other side of the surface u' after refraction at the surface. This information is then fed into the paraxial transfer equation to determine the ray height at a surface a distance t' away.

Exercise 1.3 Small-angle approximation
At what angles, expressed in both radians and degrees, are the values of the sin(u) and tan(u) functions within 1% of their angles?

If there were more than one surface in the lens system, given the ray height y' and ray angle at the second surface u', the paraxial refraction equation can be used again to calculate the ray angle after refraction at the second surface followed by a transfer to the next surface, and so on until the ray is traced through all of the surfaces in the optical system.

If that's all there is to ray tracing, what's the big deal? For one thing, not all ray angles are small, and not all surfaces are spherical. Therefore, the computation needed to *rapidly* trace a ray through multiple surfaces with a high degree of accuracy requires well-written software running on a high-performance computer. Furthermore, a ray tracing program allows us to improve a design by modifying its curvatures, thicknesses, and glass types.

1.2 Lenses

Although optical systems may be composed of numerous lenses, mirrors, filters, and other components, we start with a single lens. It can be used to define and illustrate many concepts that will be applied to more elaborate systems. We begin by using the terms defined in the box, "Anatomy of a Simple Lens," to describe the passage of light rays though the lens. For example, the optical axis, the axis of symmetry through the center of a lens, provides a line of reference for our simple optical system, a single lens.

Light from a faraway source in front of the lens, made up of rays parallel to the optical axis, will be focused to a point behind the lens on the optical axis, the focal point F′ by a positive lens (Fig. 1.9(a)), or will appear to diverge from a point in front of the lens on the optical axis from a focal point F by a negative lens (Fig. 1.9(b)). (For a discussion of what constitutes a faraway source and how it is treated in optical design, see the box, "Far Away," at the end of Section 2.1). This is the focal point of the lens. The distance between the lens and its focal point f is the focal length of the lens.

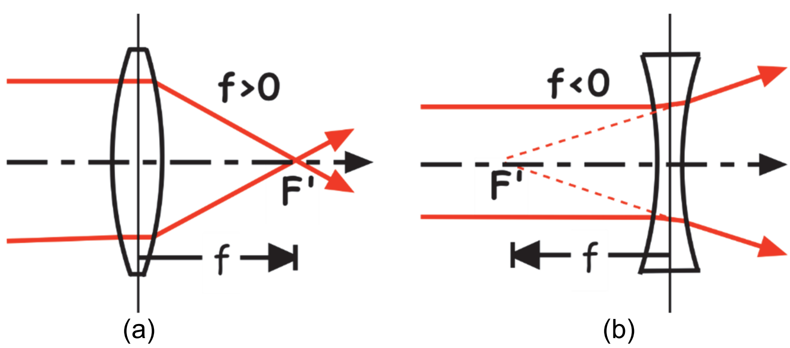
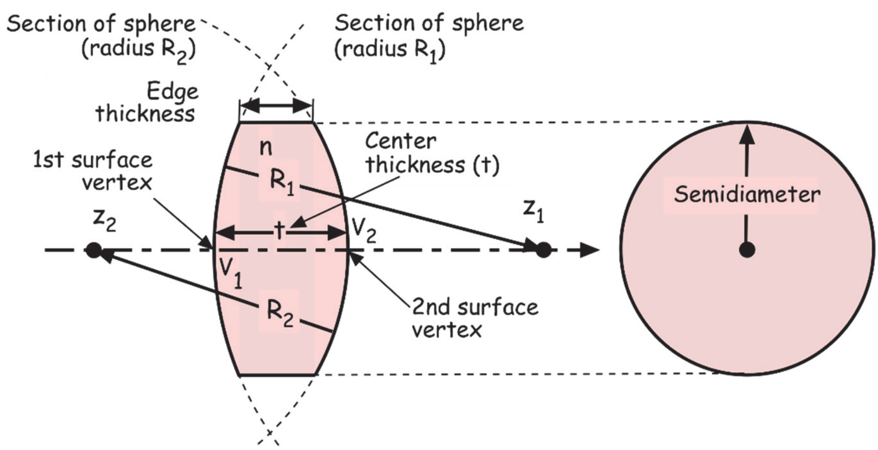


Figure 1.9 Parallel rays (a) focused by a positive lens and (b) diverged by a negative lens.

Anatomy of a Simple Lens

Most everyone knows what a lens is. Some of us know how it functions. But few people can describe one in any detail. To be able to understand how it works and to be able to modify its construction, a single lens should be described so that its performance and ailments (aberrations) can be diagnosed. That being the case, we provide, herewith, the anatomy of a single lens. It will provide a glossary of terms that will be used for the rest of the text.



A simple lens consists of a piece of glass shaped by two opposing surfaces, each being a section of a sphere. The shape of the first surface is defined by its **radius of curvature** R_1, which is the radius of the sphere whose **center of curvature** is located at z_1. The second surface has a radius of curvature R_2 with a center of curvature at z_2.

A line through the two centers of curvature defines the **optical axis** of the lens, and the separation between the two surfaces along the optical axis is the **center thickness** of the lens t. The point where the optical axis intersects a surface is the **vertex** for that surface. The two vertices are labeled V_1 and V_2 in the above figure.

The size of the lens is specified by its **semi-diameter**, shown in the right figure. To avoid any confusion with the radius of curvature, we will use the term "semi-diameter" for the lens rather than the radius. Once the semi-diameter is given, the **edge thickness** of the lens is determined, as can be seen in the left figure.

Not all lens surfaces are spherical. Some are parts of conic surfaces or more elaborate geometrical functions. However, the radius of curvature measured at the vertex of the surface provides a base radius of curvature.

In practice, the focal length of the lens f is not as well defined as it is shown in Fig. 1.9. Although the focal point would seem easy to locate (not so, as we'll see later), the plane that represents the lens cannot be defined without further analysis. But for an initial design or for a preliminary layout, it is easiest to use the **thin lens approximation**. In this case, the lens is treated as a thin optical component located on a plane in the middle of the actual lens. One definition of a thin lens is a lens whose thickness is smaller (say, one-tenth) than its focal length.

In Section 1.1, we discussed the optical power of a surface (Eq. (1.7)). We expect that each of the surfaces in a lens to contribute to the overall power of the lens. Because we are ignoring its thickness, the power of the thin lens ϕ is simply the sum of the surface powers (Eq. (1.7)), or

$$\phi = \phi_1 + \phi_2 = \frac{(n-1)}{R_1} + \frac{(1-n)}{R_2} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right). \tag{1.8}$$

The result is that the power of a thin lens is dependent only on the radii of curvature of its two surfaces (R_1 and R_2) and its refractive index n. These variables are shown in Fig. 1.10. The subscripts of the radii of curvature are assigned to the variables in order from left to right, the direction of the light entering the system or, in this case, lens. Each of these is a directed distance and is depicted as an arrow in Fig. 1.10. The radius of curvature R_1 is a **positive** quantity because its center of curvature z_1 is to the **right** of the first lens surface. R_2 is **negative** because its center of curvature z_2 is to the **left** of the second surface.

The optical power of a lens is the reciprocal of the lens focal length $\phi = 1/f$. (This definition is valid unless the object or image space medium is not air. Special cases, such as a lens immersed in a liquid or a reflective optical element, will be treated later.) The unit of power is the diopter, which is the reciprocal of the focal length when it is expressed in meters. Thus, a +100-mm- (or 0.1-m-) focal-length lens has a power of 10 diopters. The result is that the focal length of a thin lens can also be calculated from the three lens variables using the **lensmaker's formula**:

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right). \tag{1.9}$$

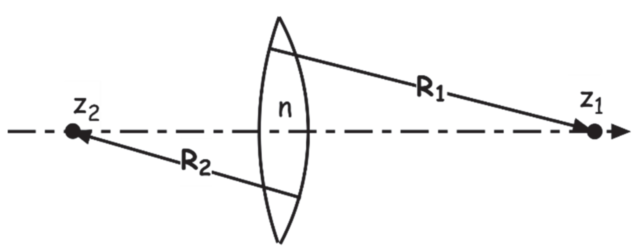


Figure 1.10 Variables that determine the focal length of a thin lens.

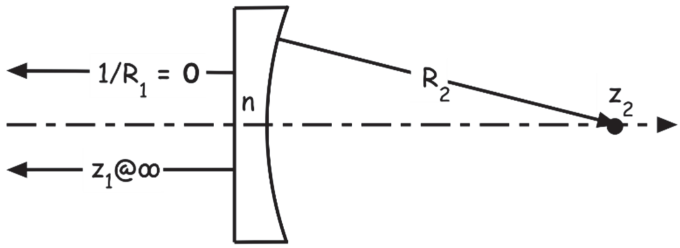


Figure 1.11 Variables for a plano-concave lens.

By substituting curvatures c for the reciprocal of the radii of curvature, the formula can be written compactly as

$$\phi = (n-1)(c_1 - c_2) = (n-1)\beta, \tag{1.10}$$

where β, the bending factor, is the difference of the curvatures $c_1 - c_2$. This form of the lensmaker's formula will be useful when we discuss the correction of color error (chromatic aberration) in a lens in Chapter 9.

For example, we can use the lensmaker's formula to calculate the focal length of the thin lens shown in Fig. 1.10 given that the refractive index of the lens is 1.5, the radius of curvature of the first surface is +150 mm, and the radius of curvature of the second surface is –100 mm. By inserting $n = 1.5$, $R_1 = 150$, and $R_2 = -100$ into the lensmaker's formula, we get

$$\frac{1}{f} = 0.5\left(\frac{1}{150} - \frac{1}{-100}\right) = \frac{1}{2}\big(0.0066 - (-0.01)\big) = \frac{0.0166}{2} = \frac{1}{120}.$$

According to the lensmaker's formula, the focal length of the lens is 120 mm.

In a second example of a plano-concave lens (Fig. 1.11), the first surface is flat. The radius of curvature of a flat surface is infinite, and the term $1/R_1$ in the lensmaker's formula equals zero. The center of curvature of the concave surface is to the right of the surface, so the radius of curvature is positive. If the refractive index of the lens is also 1.5, and the radius of curvature of the concave surface is +100 mm, then the focal length of this lens is

$$\frac{1}{f} = 0.5\left(0 - \frac{1}{+100}\right) = \frac{1}{2}(-0.01) = \frac{-0.01}{2} = \frac{1}{-200}.$$

This is a negative lens with a focal length of –200 mm. Note that the thickness of a negative lens is smaller at its center than at its edges compared to the positive lens in Fig. 1.10, where the opposite is true.

1.3 Imaging

Besides focusing parallel light to a point, a lens can collect light from many points on an object and focus these points on a plane to create an image of that object (Fig. 1.12).

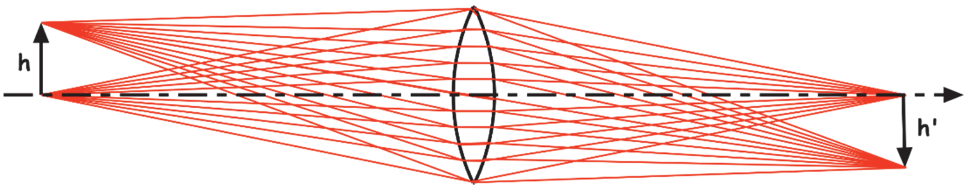


Figure 1.12 Imaging of an object with ray bundles.

In terms of imaging, the thin lens equation is the corresponding equation to the lensmaker's formula:

$$\frac{1}{t'} = \frac{1}{f} + \frac{1}{t}. \tag{1.11}$$

It can be used to find the location of the image a distance t' from the lens, for a lens of focal length f and an object located a distance t from the lens. For example, consider an object located 150 mm in front of a 100-mm-focal-length lens. By using Eq. (1.11), we insert the focal length and object distance:

$$\frac{1}{t'} = \frac{1}{100} + \frac{1}{(-150)} = \frac{1}{100} - \frac{1}{150} = \frac{3}{300} - \frac{2}{300} = \frac{1}{300},$$

and the result is that the object will be imaged at a distance 300 mm beyond the lens. Note that the object distance was entered as a negative quantity (–150) because of our sign convention (the object is to left of the lens). This may be different from the relation that you learned in sophomore physics. There, the equation was given as

$$\frac{1}{o} + \frac{1}{i} = \frac{1}{f}, \tag{1.12}$$

where o was the object distance, and i was the image distance. The discrepancy between these two equations is due to the object distance in the second equation o is considered a positive quantity, whereas the object distance t is a negative quantity because the origin of the coordinate system is located at the lens, as shown in Fig. 1.13.

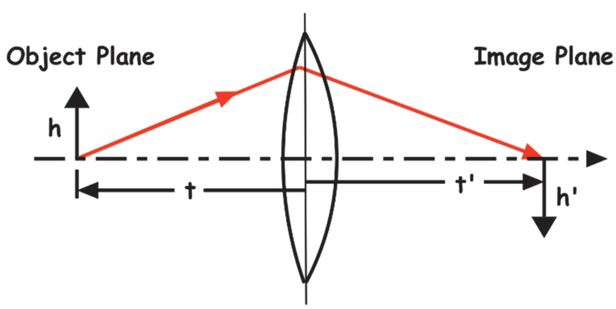


Figure 1.13 Imaging of an object by a lens of focal length *f*.

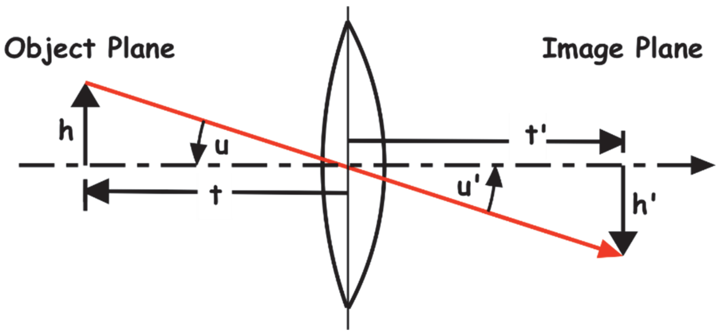


Figure 1.14 Magnification of a lens.

Now that we know how to find the location of the image, it would be nice to be able to determine its size. To start, we examine the point in the object plane that is at the top of the arrow and represents an object point at a distance h from the optical axis. The bundle of rays emitted from that point, traced in Fig. 1.12, is focused by the lens to a point on the image plane at a point h' from the optical axis. We can pick a specific ray from this bundle, one through the center of the lens that is undeviated by the lens (called a **center ray**), as shown in Fig. 1.14, and trace it to the image plane.

The two triangles in the figure have equal interior angles $u = u'$, so the tangents of these angles are equal:

$$\tan u = \frac{h}{t} = \tan u' = \frac{h'}{t'}.$$

The magnification of the optical system m is the ratio of the image height h' divided by the object height h. Solving for this ratio h'/h produces the law of magnification:

$$m = \frac{h'}{h} = \frac{t'}{t}. \tag{1.13}$$

For our example, the magnification m = 200 mm / –150 mm, or –4/3X magnification. The negative sign indicates that the image is inverted in relation to the object.

Although Eq. (1.13) is a simple equation, it establishes significant limits in the design of an optical system. The simplest application of the law shows that it is impossible to locate a magnified real image close to an imaging lens.

The definition of a center ray whose direction is undeviated by the lens is based on the assumption that the lens, however thick it may be drawn in a figure, is considered a thin lens whose front and back surfaces nearly touch. Because the points where the surfaces will nearly touch are on the optical axis, a ray aimed at that point will necessarily pass through the center of the lens. In that region, the tangents to the two surfaces are parallel to each other, so the center of the lens appears to be a parallel slab, and a ray entering the slab at some angle will emerge on the other side at the same angle, i.e., the ray is undeviated (see Ex. 1.1).

Exercise 1.4 Thin lens equation and magnification
In Section 1.3, the object was located 150 mm in front of the 100-mm-focal-length thin lens. Where will the image be located if the object is relocated to a point 200 mm in front of the lens? What will be the new magnification?

1.4 Types of Images

When an object is located outside the focal point of a positive lens, as illustrated in Fig. 1.15(a), the rays from each object point *converge* to its corresponding image point resulting in a **real image** of the object. If a white surface is placed behind the lens in the image plane, a real image can be seen there, just as we see an image projected on a screen in a movie theater. There are other optical arrangements where the lens doesn't focus light rays from an object. Instead, the rays from an object point diverge as they emerge from the lens. For example, when an object is located inside the front focal point of a positive lens, the rays diverge, as shown in Fig. 1.15(b). The image plane is located by tracing the diverging rays backwards to where they meet at a point. In this case, a screen located behind the lens will not show any image. You need to use another optical system, such as the eye, to see the image. This type of image is called a **virtual image**. In Fig. 1.15(b), the lens acts as a magnifier, displaying a larger image h' than the original object h. In comparing the lens systems in Fig. 1.15, we see that for a positive lens the real image space is on the image side of the lens (positive value for t'), whereas the virtual image space is on the object side of the lens (negative value for t'), which means that the sign of the image distance indicates whether the image produced by the lens is real (positive t') or virtual (negative t').

All of the quantities that have been defined and discussed are signed quantities. In the case shown in Fig. 1.13, the image height is inverted relative to the object, and the sign of the ratio of the heights (h'/h) is negative, as is m. The ratio of the distances gives the correct sign of the magnification because t' is positive and t is negative, as we noted earlier (Eq. (1.13)).

This introduction to ray tracing has established some of the basic concepts needed to understand and analyze optical systems. But before showing you how to enter a system into a ray tracing program, in the next chapter we want describe a graphic technique that we call ray sketching that can give you a feel for a system before any keys are pressed and routines are run.

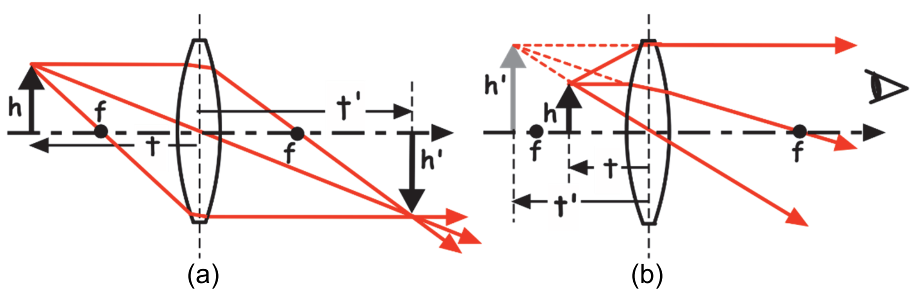


Figure 1.15 Types of images: (a) real and (b) virtual (t' is a negative quantity).

Exercises

Exercise 1.5 Equiconvex lens
$n = 1.5$ units; $t = -400$ units; $R_1 = +100$ and $R_2 = -100$. Determine its focal length f, the location of the image t', and the magnification of the system m, and indicate if the resulting image is R (real) or V (virtual). Sketch the shape of the lens.

Exercise 1.6 Biconvex lens
$n = 1.4$ units; $t = -200$ units; $R_1 = +200$ and $R_2 = -100$. Determine its focal length f, the location of the image t', and the magnification of the system m, and indicate if the resulting image is R (real) or V (virtual). Sketch the shape of the lens.

Exercise 1.7 Positive meniscus lens
$f = 266.66$ units; $t = -200$ units; $R_1 = +100$ and $R_2 = +200$. Determine the refractive index of the lens n, the location of the image t', and the magnification of the system m, and indicate if the resulting image is R (real) or V (virtual). Sketch the shape of the lens.

Exercise 1.8 Negative meniscus lens
$n = 1.5$ units; $t = -200$ units; $R_1 = +200$ and $R_2 = +100$. Determine its optical power ϕ in diopters, the location of the image t', and the magnification of the system m, and indicate if the resulting image is R (real) or V (virtual). Sketch the shape of the lens.

Exercise 1.9 Biconcave lens
$n = 1.5$ units; $t = -400$ units; $R_1 = -75$ and $R_2 = +300$. Determine its focal length f, the location of the image t', and the magnification of the system m, and indicate if the resulting image is R (real) or V (virtual). Sketch the shape of the lens.

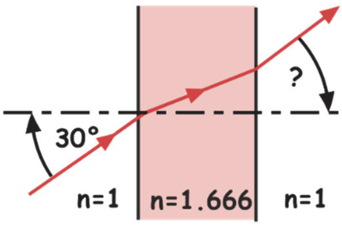
Exercise 1.10 Equiconcave lens
$\phi = -0.01333$ diopters; $t = -400$ units; $R_1 = -100$ and $R_2 = +100$. Determine the refractive index of the lens n, the location of the image t', and the magnification of the system m, and indicate if the resulting image is R (real) or V (virtual). Sketch the shape of the lens.

Exercise 1.11 A rule for lenses
Based on an observation of the **shapes** of lenses in the Exs. 1.5–1.10, how could you determine whether a lens on a laboratory table were positive or negative by just picking it up, even before you looked through the lens?

Exercise 1.12 Transfer equation II
A ray is traveling along the optical axis at a height of 10 mm above the coordinate origin with a slope angle of +0.2 radians. Where does the ray cross the optical axis?

Answers

Ex. 1.1



The index-sine product is the same for all three media, so the exit angle equals the entrance angle, 30°.

Ex. 1.2

$y' = y + t' \tan u' = 10 + 50 \tan(0.2)$
$= 10 + 10.14 = 20.14;$
$y' = y + t' \tan u',$
$75 = 10 + t' \tan(0.2),$
$65 = t' \cdot 0.203,$
solving for $t' = 65/0.203 = 320.66$ mm.

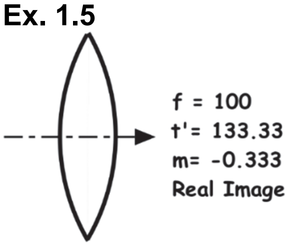
Ex. 1.3

Depending on round-off, the value of the angle or the sine is 0.24 rad, or 14°. For the tangent, i = 0.17 rad, or 10°.

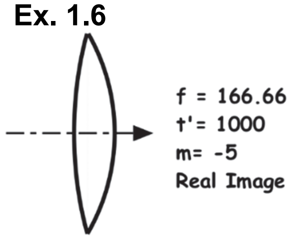
Ex. 1.4

The image is a real image located 200 mm behind the lens. Its magnification $m = t'/t = -200/200 = -1$. The image is inverted and the same size as the object.

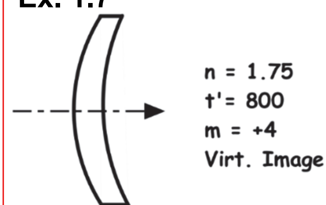
Ex. 1.5



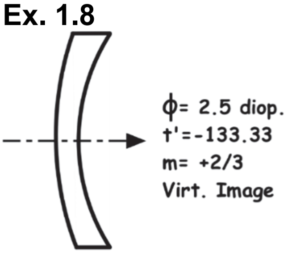
Ex. 1.6



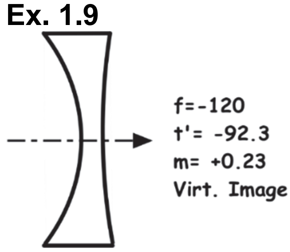
Ex. 1.7



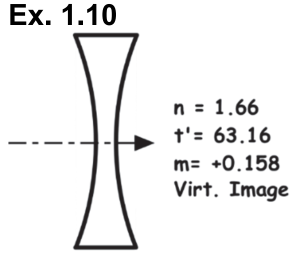
Ex. 1.8



Ex. 1.9



Ex. 1.10



Ex. 1.11

The center thickness of a positive lens is larger than its edge thickness, whereas for a negative lens the reverse is true.

Ex. 1.12

Solving $0 = 10 + t \tan(0.2)$, the ray is traveling upward and had already crossed the optical axis at –49.33 mm.