2.3 Concept of Active Acoustic Metamaterials

In order to understand the concept and the structure of an active acoustic metamaterial, consider the structure of the external and internal acoustic cloaks (Fig. 2.3), beam shifters (Fig. 2.4), and beam focusing device (Fig. 2.5). In Fig. 2.3, the external cloaks can potentially be used to hide acoustically critical objects, whereas the internal cloak can be employed to render the fuselages of aircraft and helicopters quiet, and improve the interior acoustics inside auditoriums and meeting rooms. Note that acoustic cloaks can be configured to treat objects either externally or internally in order to render these objects acoustically invisible. As shown in Fig. 2.4(a), beam shifters can be utilized to enable control of the directivity and dispersion characteristics of the waves as these waves propagate inside the metamaterial, in order to shift the direction and decrease/increase the sound propagation speed. A possible application can be designing a metamaterial helmet for soldiers and athletes [Fig. 2.4(b)]. The helmets are intended to protect the soldiers and athletes from any brain injuries resulting from exposure to a blast or impact wave
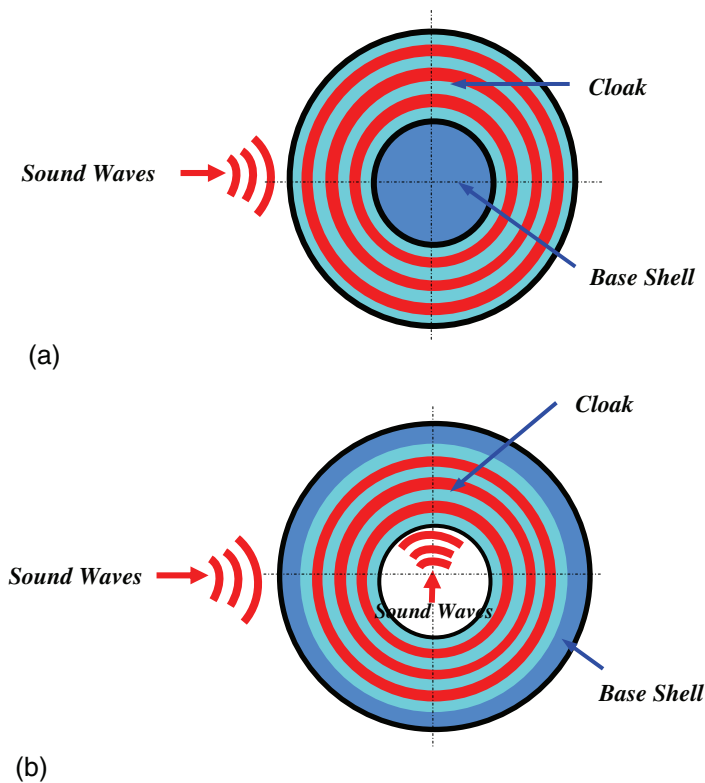

Figure 2.3 External and internal acoustic cloaks: (a) external cloak (submarines, torpedoes, columns in auditorium, and so on); and (b) internal cloak (fuselage of aircraft, helicopter cabins, auditorium, and so on).[23]
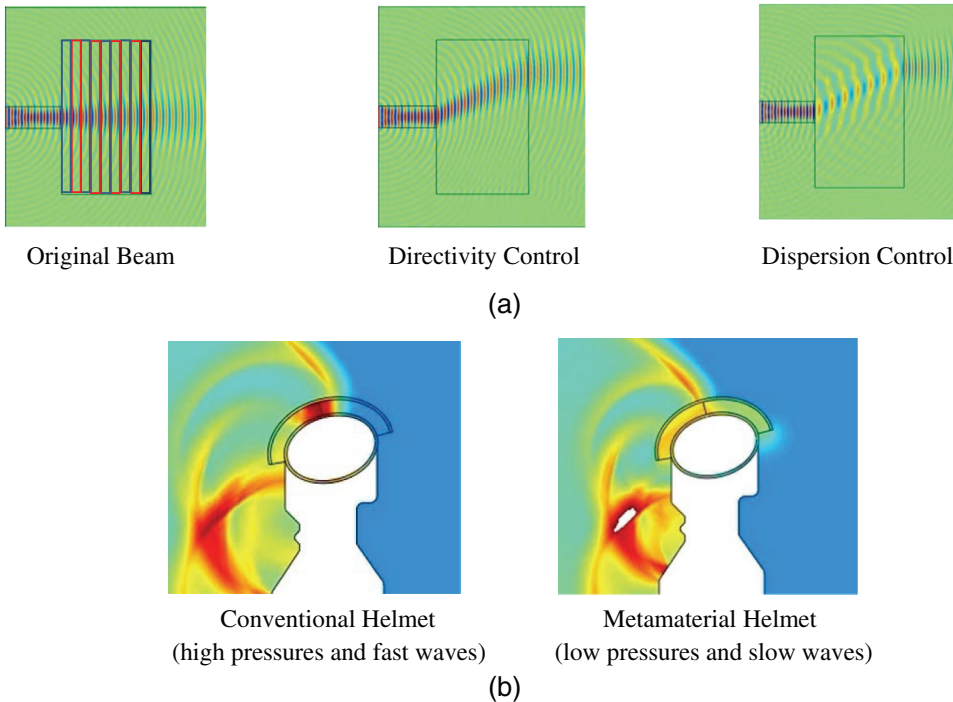
Original Beam   Directivity Control   Dispersion Control

(a)

Conventional Helmet   Metamaterial Helmet
(high pressures and fast waves)   (low pressures and slow waves)

(b)

Figure 2.4 Beam shifting metamaterial devices: (a) beam shifters (from left to right: original beam control, directivity control, dispersion control); and (b) helmets [left: conventional helmet (high pressures and fast waves); right: metamaterial helmet (low pressures and slow waves)].

propagation, by smoothly guiding and bending these waves around their heads to avoid any over-pressurization. Figure 2.5 shows a schematic of a metamaterial acoustic beam focusing device that can be useful to manufacture, for example, hearing aids with a high signal-to-noise ratio or for enhancing the efficiency of acoustic energy harvesters.

Operation of the systems and devices shown in Figs. 2.3–2.5 rely on engineered materials comprised of periodically arranged composite cells of two or more materials, which are arranged in such a way to yield effective density and bulk modulus distributions that are otherwise unachievable with isotropic materials. The unit cell in all of these examples is shown in Fig. 2.6. Note that the dimensions of these unit cells are designed to be much smaller than the wavelength of the encountered propagating waves in order to achieve a homogenized domain with the targeted overall material properties. The parameters in designing the cell itself are its geometry, orientation in the macroscale structure, as well as material composition.

The preferred arrangement of the unit cell of the active acoustic metamaterial, shown in Fig. 2.6, consists of arrays of fluid cavities that are
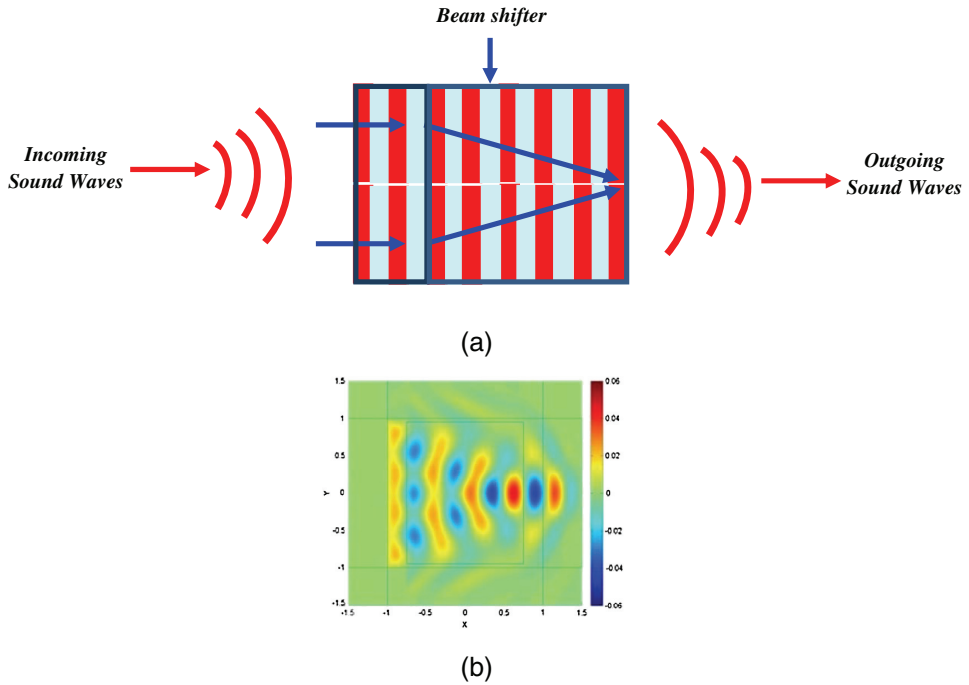
(a)

(b)

Figure 2.5 Metamaterial acoustic beam focusing device: (a) schematic; and (b) focusing capabilities of the device.
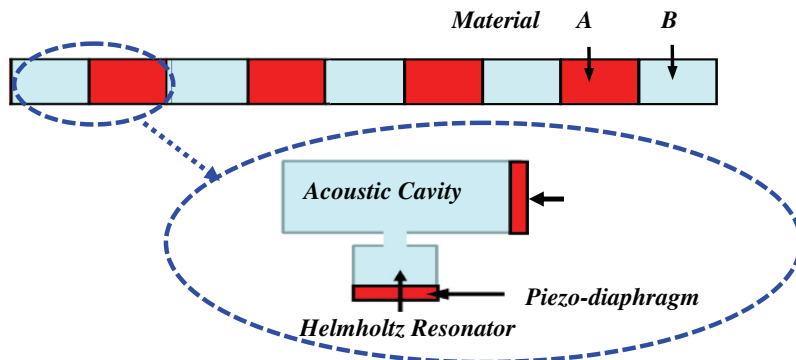

Figure 2.6 Structure of an active acoustic metamaterial.

separated by and provided with piezoelectric boundaries. These boundaries control the stiffness of the individual cavities and in turn control the cavity dynamical density and bulk modulus. All of the cavities and boundaries appear mechanically the same to facilitate manufacturing. However, with various control strategies, the individual piezo-boundaries can be programmed to achieve any desirable spectral and spatial distributions of the acoustical density and bulk modulus over the metamaterial volume.

2.4 Dynamic and Control Characteristics of a Metamaterial Unit Cell with Programmable Density and Bulk Modulus

2.4.1 Overview

In this section, the basic equations governing the dynamics and control of a unit cell of a metamaterial are presented. The considered unit cell has programmable capabilities for simultaneously controlling the effective bulk modulus and density.[24–30] A schematic of the unit cell is shown in Fig. 2.7 along with its main geometrical and physical parameters.

An acoustic cavity with uniform cross-sectional area A and length $2 \times l_f$, subject to an acoustic pressure drop Δp resulting in volumetric flow rate Q, is considered. The fluid inside the cavity is characterized with static fluid density ρ_f and bulk modulus B_f. The value of Q depends on three major forces: inertia forces due to the mass of the fluid inside the cavity; elastic forces due to the "stiffness" of the entrapped fluid volume; and friction and damping forces, which are ignored in the current analysis.

2.4.2 Model

Passive metamaterial cell

Newton's second law necessitates that the net force due to the pressure difference Δp equals the rate of change of the fluid momentum, as given by

$$A\Delta p = \rho_f l_f A \frac{dv}{dt},$$

(2.1)

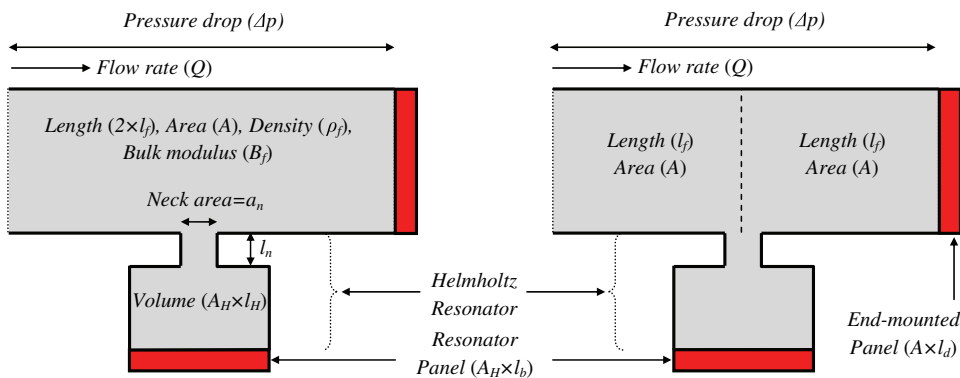where v is the particle velocity.

Figure 2.7   Schematic of the developed cell for density and bulk modulus control.[26]

For a uniform velocity along the cavity length, $\frac{dQ}{dt} = A\frac{dv}{dt}$. Rewriting Eq. (2.1) yields

$$A\Delta P(s) = \rho_f l_f s Q(s), \tag{2.2}$$

where s is the Laplace complex number. Hence,

$$\frac{\Delta P(s)}{Q(s)} = \frac{\rho_f l_f}{A} s = L_F s = Z_{LF}(s), \tag{2.3}$$

where $Z_{LF}(s)$ denotes the inertial impedance of the cavity and is equal to the amount of pressure difference needed in order to drive a unit volume flow rate of the fluid inside the cavity, which is proportional to the density of the fluid and the length of the cavity, and inversely proportional to its cross-sectional area.

The second type of forces is due to the "stiffness" of the fluid inside. A pressure drop of Δp across the cavity will cause the fluid to be "strained" due to a change in its volume Vol by an amount $d(Vol)$, as follows:

$$\varepsilon_V = -\frac{d(Vol)}{Vol}. \tag{2.4}$$

Since the bulk modulus B_f is defined as the difference in pressure that yields a unit volume strain, Δp is defined as

$$\Delta p = -B_f \frac{d(Vol)}{Vol}. \tag{2.5}$$

The change in volume $d(Vol)$ along a period of time dt is defined as $d(Vol) = -\int Q dt$. Hence, applying Laplace transformation to Eq. (2.5) results in

$$\Delta P(s) = \frac{Q}{s} \frac{B_f}{Al_f}. \tag{2.6}$$

Consequently, the elastic impedance $Z_{CF}(s)$ is

$$Z_{CF}(s) = \frac{\Delta P(s)}{Q(s)} = \frac{1}{C_F s} = \frac{1}{s} \frac{B_f}{Al_f}. \tag{2.7}$$

The total impedance of the straight cavity is therefore given as

$$Z_t(s) = Z_{LF}(s) + Z_{CF}(s) = \frac{\rho_f l_f s}{A} + \frac{B_f}{Al_f s}. \tag{2.8}$$

Utilizing the electroacoustic analogy, the Helmholtz resonator is introduced, adding to the total impedance of the proposed metamaterial cell. The mechanical stiffness and mass of the flexible panel coupled to the acoustic cavity are modeled using an ideal transformer, which in this case couples the mechanical and acoustical domains, where the transformation ratio is the

surface area of the flexible panel. This is valid for a flexible panel mounted at the end of the fluid cell as well as that at the end of the Helmholtz resonator. The components in the mechanical side of the transformer are transferred to the acoustical side taking the transformation ratio into consideration. In the mechanical side, the current represents the mechanical velocity $\dot{x}$, the voltage drop represents the mechanical force F, and the system spring and mass are represented by a capacitor $C_D = 1/K_D$ and inductor $L_D = M_D$ in the electrical circuit, respectively. From Eq. (2.8), the acoustic mass and stiffness in each cavity section is represented by the inductor $L_F = \rho_f l_f / A$ and capacitor $C_F = A l_f / B_f$, respectively. The Helmholtz resonator is also represented as the fluid inductance L_H, fluid capacitance C_H coupled with the inductance L_B, and capacitance C_B of the Helmholtz resonator flexible panel.

Arranging all of the inductances and capacitances into one electrical circuit introduces the Helmholtz resonator mass $M_H = \rho_f \times l \times a$, stiffness $K_H = B_f a^2 / (A_H \times l_H)$, a is the Helmholtz resonator's neck cross-sectional area, l is its neck length, and A_H and l_H are the resonator cavity's cross-sectional area and length, respectively.

Therefore, the effective density and bulk modulus for the cavity can be calculated by considering the volumetric rate of change of the acoustic cavity, given as

$$\dot{p} = -B_e \frac{Q - Q_H}{2Al_f}, \tag{2.9}$$

$$\frac{\Delta p}{2l_f} = -\rho_e \frac{Q}{A_f}, \tag{2.10}$$

where B_e and ρ_e are the effective bulk modulus and density, respectively.
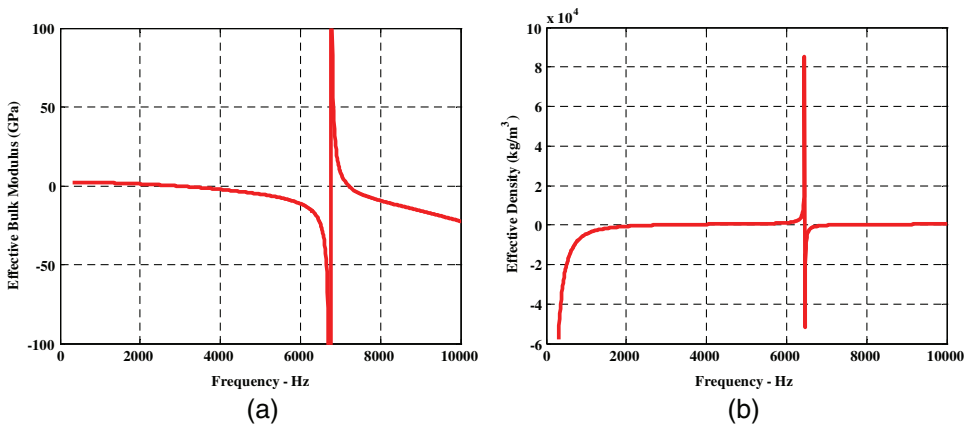

Figure 2.8 (a) Uncontrolled homogenized bulk modulus and (b) density for a cavity a with Helmholtz resonator.