which we designate as Y. The value of $\sqrt{\pi}$ is the flaw shape factor of a Griffith crack, i.e., where b/c approaches the infinite:

$$Y = \sqrt{\pi}. \tag{2.5}$$

This is the most severe shape of flaws, and $Y = 1.77$.

2.4.1 Finite bodies and free-surface correction

The Griffith formulation of shape factor applies to through flaws that are embedded in an infinite body, as has been shown in Fig. 2.1. Many handbooks[4] and texts show modifications to the shape factor when the body in which the crack is embedded is not infinite. This is depicted in Fig. 2.3. Such modifications are dependent on the flaw-depth-to-body-thickness ratio, as shown in Fig. 2.4. The higher the ratio, the higher the stress intensity amplification.

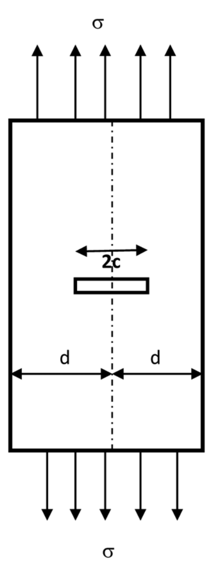

Figure 2.3 Embedded crack in a finite body.

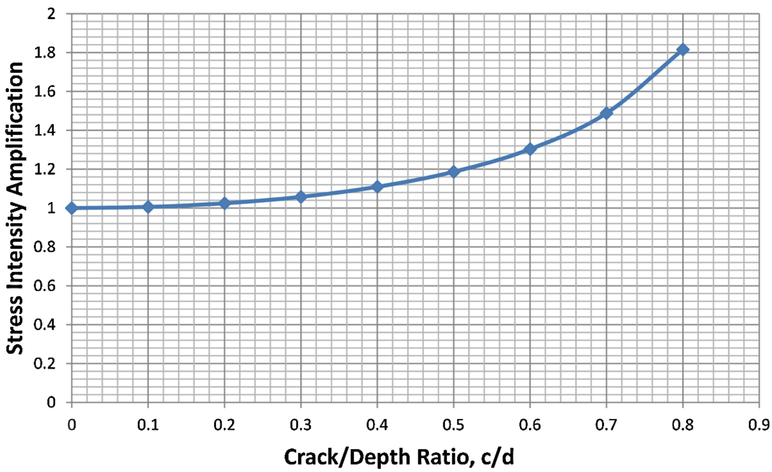

Figure 2.4 Embedded crack stress intensity versus crack-depth-to-body-thickness ratio.

However, most of our interest is in surface flaws, as in Fig. 2.2. Further, most of our concerns are for flaws that are much smaller than the material thickness in which they exist; i.e., the flaw-depth-to-material-thickness ratio approaches zero. In this case, we find a free-surface correction factor that can be obtained from theoretical Laurent series expansion formulation with appropriate boundary techniques.[5] Suffice it to say that, due to an increase in strain energy at the boundary, the value of the free-surface correction factor, which is to be multiplied by Y, is given as 1.122, when c is much less than the specimen depth.

A plot of the free-surface correction factor versus flaw-size-to-body-thickness ratio is given in Fig. 2.5, super-imposed on the embedded crack modification. The effect of

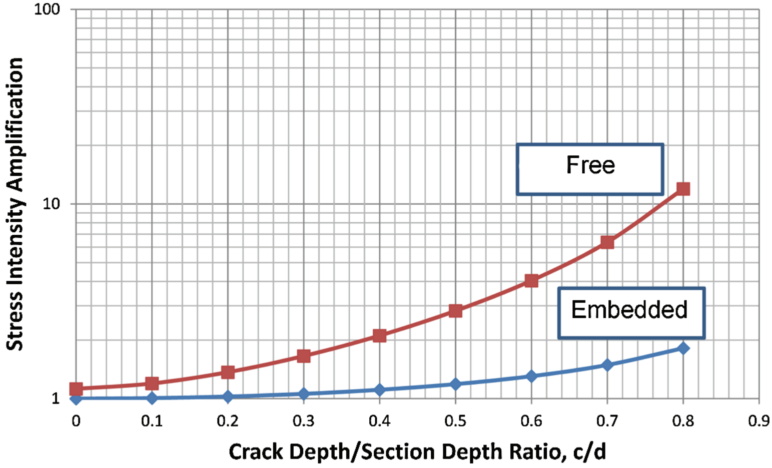

Figure 2.5 Free surface versus embedded-crack stress intensity amplification.

the free surface is more pronounced for higher crack-depth-size-to-specimen-thickness ratios. However, glass and ceramic flaws are generally much smaller than the body thickness; a large, 300-μm surface flaw (0.012 in.) in a 1-in.-thick component results in a flaw-to-depth ratio of 0.012. An expanded view in Fig. 2.6 shows the effect to be rather inconsequential relative to the usual assumption of c/d being near zero. Thus, in general, for a free surface, we find the Griffith shape factor to be

$$Y = 1.122\sqrt{\pi} = 1.98. \tag{2.6}$$

2.4.2 General point flaws

Most flaws are not Griffith flaws; i.e., they are not through flaws ($b/c = \infty$) but are point flaws (e.g., the penny crack, where $b/c = 1$). We can solve for the value of Y by using advanced

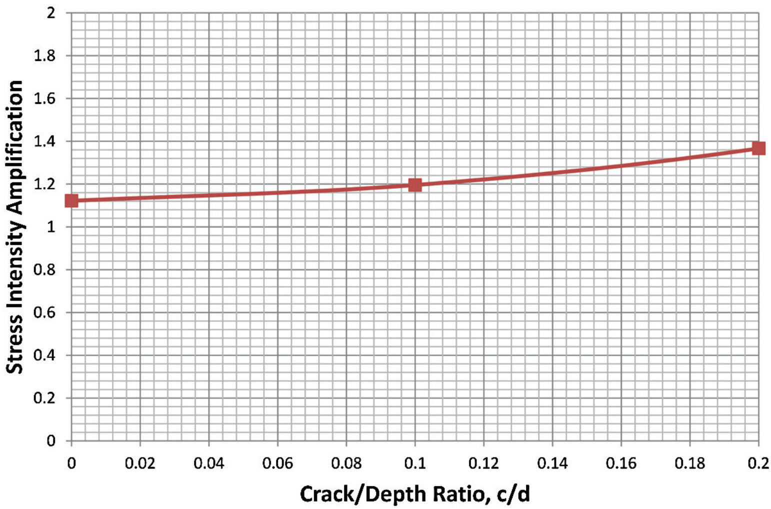

Figure 2.6 Free-surface correction factor versus crack-to-body-depth ratio.

fracture mechanics techniques. This is given[6] as an elliptical integral of the second kind, with free-surface correction, as

$$Y = \frac{1.12\sqrt{\pi}}{\phi}, \tag{2.7}$$

where

$$\phi = \int_0^{\pi/2} \left[\cos^2\theta + \left(\frac{c^2}{b^2}\right)\sin^2\theta\right]^{1/2} d\theta \;\; (c \le b), \tag{2.7a}$$

$$\phi = \left(\frac{c}{b}\right)\int_0^{\pi/2} \left[\cos^2\theta + \left(\frac{c^2}{b^2}\right)\sin^2\theta\right]^{1/2} d\theta \;\; (c \ge b). \tag{2.7b}$$

The integral is carried over the half-angle subtended by the elliptical shape of the crack, as shown in Fig. 2.7.

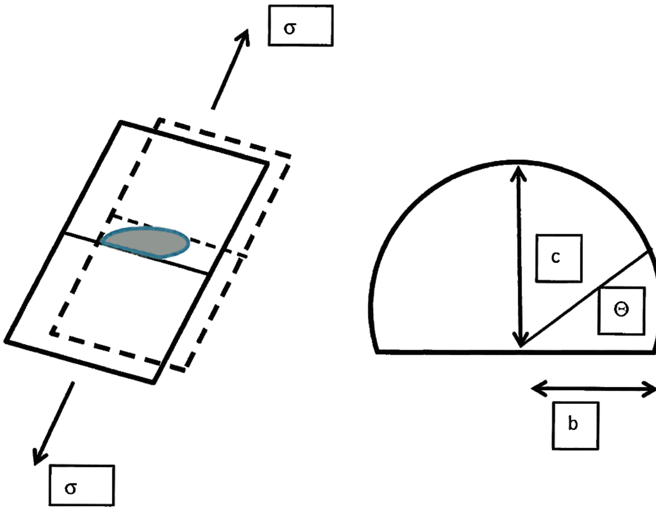

Figure 2.7 Flaw shape factor is a function of the integral over the flaw half-angle.

The integral is readily solved for the Griffith and half-penny flaws but not so readily without the use of complex calculus. Fortunately, the calculus has been done for you. Table 2.1 gives the Griffith and penny solutions, while Fig. 2.8 shows the entire domain, in which a surface crack is illustrated. In most instances, the value of Y ranges from 1.0 to 1.98, with the half-penny crack solution giving a value of 1.26.

Table 2.1 Flaw shape factor Y for Griffith and penny cracks (b = flaw half-width; c = flaw depth).

Crack type	*b*/*c*	ϕ	*Y*		
			Internal flaw	**Free-surface correction**	**Surface flaw**
Griffith	>10	1	$\pi^{1/2}$	1.12	1.98
Penny	1	π/2	1.13	1.12	1.26