Teaching quantum electronics to Electronic Engineering undergraduates

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ABSTRACT

In this paper the construction is described of an undergraduate course in Quantum Electronics and Lasers which has been presented to Third Year students of Electronics and Electrical Engineering in the University of Glasgow each year since 1995. When expressed in matrix form the quantum mechanics of the 2-level atomic system is quite accessible to the average student of Electronic Engineering, since it involves mathematics identical to that already used in simple circuit theory, namely systems of a finite number of linear ordinary differential equations.

Keywords: quantum electronics, lasers

1. INTRODUCTION

The proper understanding of the behaviour of lasers requires a certain element of understanding of the principles of quantum mechanics. It is widely believed that it is extremely difficult to teach the elements of quantum mechanics to undergraduates in Electronic and Electrical Engineering courses because they do not usually have the appropriate physics background in atomic spectroscopy, Hamiltonian mechanics, etc., nor the mathematics that is associated with that viewpoint, namely partial differential equations, integrability, etc.

The understanding of lasers requires little more than the dynamics of a 2-level atomic system, coupled to a time-varying electric field. Several very successful laser physics text books already take this point of view.[1–3] These books usually adopt as a starting point the conventional physics formulation of quantum mechanics based on the Schrödinger equation for the 1-electron atom in *position representation*, the 2-level system being obtained as a by-product after many approximations from the full atomic dynamics. This approach axiomatically requires formulation using partial differential equations, solutions by separation of variables, the theory of linear operators, and a lot of other mathematical machinery which is generally not possessed by most Electronic Engineering students at the stage where we wish to present a basic course in quantum electronics.

On the other hand, when expressed in *matrix* form, the quantum mechanics of the 2-level atomic system is quite accessible to the average student of Electronic Engineering, since it involves mathematics identical to that already used in simple circuit theory, namely systems of a finite number of linear ordinary differential equations. No partial differential equations are involved, and it is possible also to make connections with simple optical phenomena which are similarly described by 2-dimensional matrix operations such as polarisation rotation, $ABCD$-matrix descriptions of ray optics and so on. There are almost no textbooks which adopt simple matrix quantum mechanics as a *primary* point of view for a reasonably complete systematic theory. One honourable exception, now long out of print, is by Troup[4]; interestingly, the author states in the Acknowledgements in this book that it was used as the basis for a course to Electrical Engineering students at Monash University in Australia in the 1960s.

In this paper I will discuss issues, some philosophcal and some educational, which arise in the construction of an undergraduate course in Quantum Electronics and Lasers. This course has actually been presented to Third Year students of Electronics and Electrical Engineering in the University of Glasgow each year since 1995.

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The basis of the theory is the 2×2 matrix system, whose time evolution is described by systems of ordinary differential equations. The exposition will be recognised as essentially that of Dirac,[5] using conventional vector-matrix notation without Dirac's bra-ket symbols; these can be easily introduced if desired. The following paper is basically a description of the main points of the development; many points have to be explained carefully in some detail, but the nature of this paper does not permit a full description of all these points.

2. FOUNDATIONS OF QUANTUM ELECTRONICS

Quantum theory involves a set of postulates, which forms the basis of a mathematical theory of arbitrary systems. Fundamentally it is asserted that the state at any time t of a physical system S is completely described by a list of complex numbers; this list of complex numbers $\{v_1, v_2, \ldots, v_n\}$ is called the *state vector* and is given the symbol v. These numbers are supposed to be internal variables of the system with a completely abstract meaning. Since it is supposed that the state of system S is evolving in time, it is natural to assume that the elements of the vector v are time dependent, $v(t)$. It is not important what the nature of the system S might be, it could equally well be the electrons in a solid, the world stock market or even the human brain. A complete specification of the state of the system at any time requires only a large enough number of elements of the state vector; in principle this number might be infinite. It is also not required that these numbers be physically observable, they are abstract internal variables of the system.

It is further supposed that this list of numbers evolves in time according to a *linear* law of evolution, written mathematically as

$$i\hbar \partial_t v = Hv \tag{1}$$

where H is some matrix (or operator), and $\hbar$ is a scaling constant chosen to have certain physical units (Joule-seconds in the case of quantum electronic systems), which are to be deduced later in the theory.

Students of Electronic Engineering will be very familiar with mathematical models of the form of eq. (1), since this is exactly the form taken by the laws of elementary circuit theory. If the vector v is made up of the (internal) nodal voltages V_j and loop currents I_k in a circuit consisting of passive linear components, then the transient behaviour of any circuit is described by a system of equations of the form eq. (1), with $\hbar = 1$ and the elements of H made up from combinations of the R, L and C values of components in the circuit. Also, state-variable descriptions of control systems using this type of mathematics may be familiar to students. Clearly, arbitrarily complicated circuits or control systems can be described in this way by a large enough state vector v. It may then be raised as a *hypothesis* that *arbitrary systems of any kind* may be described by some model of the form of eq. (1) with a large enough state vector.

Of course this model of a system is very abstract, and it is necessary to explain how *observations* or *measurements* of properties of the system can be related to the abstract variables in the state-vector. For this purpose, it is noted that any measurement of the system will change its state, and can therefore in principle be described by a matrix which maps the state vector before the measurement to the state vector after the measurement, a transformation of the form $v \to Av$, with each possible physical measurement of the system being described by a corresponding matrix for A. Thus the measurement is represented by the effect that it has on all possible configurations of the state vector. Since, however, the state vector is not itself observable, this cannot constitute the *result* of a measurement. The result of a measurement must be a single real number which depends on both the state v of the system at the time that the measurement was performed, and on the particular measurement carried out, i.e. the particular matrix A. This leads, after the definition of the inner product $\langle w, v \rangle$ of two state-vectors, to the concept of the *expectation* $\langle A \rangle = \langle v, Av \rangle$, which is precisely a single number depending on both the state of the system and the particular measurement performed. This number can be guaranteed to be real if the matrix A is *Hermitian*. From these definitions, the fundamental quantum-mechanical relation for the evolution of observables

$$\partial_t \langle A \rangle = (i\hbar)^{-1} \langle [A, H] \rangle \tag{2}$$

is constructed, where [,] represents the *commutator* of two matrices.

Physical laws of the classical kind are generally differential equations connecting the results of observable quantities. Eq. (2) is of exactly this form. It is important to note that the state vector v has completely disappeared from eq. (2), and the equation refers only to observable quantities.

Some simple deductions are possible from eq. (2). First, suppose that the observable A is taken to be the identity matrix, I; since the identity commutes with all matrices, the content of eq. (2) is just that $\langle I \rangle$ is constant in time, which in turn is equivalent to the conservation of the *norm* of the state vector: $\langle v, v \rangle = \langle v, Iv \rangle = \langle I \rangle =$ constant. Further, if A is taken to be the system matrix H, then H commutes with itself and eq. (2) says that $\langle H \rangle =$ constant. Thus, for any system described by this formalism it is necessary that the observable H has a constant expectation value. The only physical candidate for a quantity that is preserved by any such system is the system *energy*, and H is accordingly interpreted as the observable energy. This means in turn that H has dimensions of energy (Joules) and the scaling constant $\hbar$ has dimensions of *action* (Joules-seconds).

It is then possible to list the fundamental postulates of quantum theory as proposed by Dirac[5]:

- the quantum system is described by a state vector v, which lives in a *Hilbert space* $\mathcal{H}$;

- there is an inner product $\langle w, v \rangle$ defined for any two vectors w and v in the Hilbert space;

- the state vector evolves according to the Schrödinger equation (1);

- observables are described by Hermitian matrices, whose expectation values are measurable quantities;

- expectations of observables evolve with time according to eq. (2).

3. 2-LEVEL QUANTUM SYSTEM

Many phenomena of interest in quantum electronics can be described by quantum systems restricted to a 2-element state vector $v = \{v_1, v_2\}$. This means that observables must be 2×2 Hermitian matrices. There are only 4 linearly independent possibilities for 2×2 Hermitian matrices, given by the three Pauli matrices $\{\sigma_j : j = 1, 2, 3\}$ in association with the 2×2 identity matrix I. The Pauli matrices satisfy the commutation relations

$$[\sigma_j, \sigma_k] = 2\mathrm{i}\varepsilon_{jkl}\sigma_l \tag{3}$$

where ε_{jkl} is equal to 1 for $\{jkl\}$ any cyclic permutation of $\{123\}$, antisymmetric with respect to interchange of indices, and 0 for any two indices identical. Any three matrices satisfying (3) will serve for the Pauli matrices. Choose for the system matrix $H = -\frac{1}{2}\hbar\Omega_0\sigma_3$; then the other two matrices satisfy

$$\begin{aligned} \partial_t\rho_1 &= \Omega_0\rho_2 \\ \partial_t\rho_2 &= -\Omega_0\rho_1 \end{aligned} \tag{4}$$

where $\rho_1 = \langle\sigma_1\rangle, \rho_2 = \langle\sigma_2\rangle, \rho_3 = \langle\sigma_3\rangle$. It is also easily proved that $\rho_1^2 + \rho_2^2 + \rho_3^2 = 1$. Eqs. (4) describe a simple harmonic oscillator. If we redefine the matrices

$$Q = q_0\sigma_1, \quad P = m\Omega_0 q_0\sigma_2 \tag{5}$$

by scaling, with a normalising length scale q_0, then (4) is equivalent to

$$\begin{aligned} \partial_t q &= \frac{p}{m} \\ \partial_t p &= -m\Omega_0^2 q \end{aligned} \tag{6}$$

where $q = \langle Q \rangle, p = \langle P \rangle$. Eqs. (6) are the equations of a particle of mass m undergoing simple harmonic motion at a natural resonant frequency Ω_0 in a quadratic potential $V = \frac{1}{2}m\Omega_0^2 q^2$ with displacement q and momentum p.

The model of a particle undergoing simple harmonic motion in a quadratic restoring potential is exactly the model of the electronic polarisation of an elementary atom described classically by Lorentz.[6] This permits the *interpretation* that the matrices Q and P are the observables in the quantum domain corresponding to position and momentum in the classical domain. These observables then satisfy the same differential equations as their classical correspondents. However, the relation between these observables and the system energy is different in

the quantum domain to that which holds in the classical domain. For the quantum case we get for the energy of the particle above the ground state

$$W = \tfrac{1}{2}\hbar\Omega_0 + \langle H \rangle = \tfrac{1}{2}\hbar\Omega_0(1 - \rho_3) = \tfrac{1}{2}\hbar\Omega_0\left(1 - \sqrt{1 - \frac{1}{q_0^2}\left(q^2 + \frac{p^2}{m^2\Omega_0^2}\right)}\right) \qquad (7)$$

whereas for the classical case we get

$$W = \frac{m\Omega_0^2 q^2}{2} + \frac{p^2}{2m}. \qquad (8)$$

These are only comparable when ρ_1 and ρ_2 are sufficiently small and

$$\tfrac{1}{2}\hbar\Omega_0 = m\Omega_0^2 q_0^2, \qquad (9)$$

which defines the length scale q_0.

4. INTERACTION OF EXTERNAL ELECTROMAGNETIC FIELD

When an atom is placed in an electromagnetic field there is an interaction between the dipole induced by the oscillations of the electron and the field. This interaction alters the energy of the atom. Let E be the applied electric field. Then the energy of the atom interacting with the field in the classical case is

$$W = W_0 + eqE \qquad (10)$$

where W_0 is the energy of the atom in the absence of electric field, $-e$ is the charge on the electron, and q is the displacement of the electron from its equilibrium position. The conversion of this expression to the corresponding quantum-mechanical version requires the replacement of the quantities W_0 and q by expectations of the appropriate matrices. For the 2-level atom we know these quantum matrices. Thus we have the quantum-mechanical matrix

$$H = -\tfrac{1}{2}\hbar\Omega_0\sigma_3 + eq_0E\sigma_1 \qquad (11)$$

to represent the interacting atom and field. This is assumed to apply even in the fully time-dependent case when E is a function of time t. It should be noted here that this expression for H does not have a time independent expectation value because it does not account for the energy which is stored in the field E. It is however possible to add a further term to H which compensates for this missing energy; since this term only depends on the field E and not on the state of the atomic system, it is represented as a matrix by $W_{\mathrm{em}}I$ where W_{em} is a scalar, and this term naturally commutes with any operator so cannot contribute anything to the quantum dynamics of the system. Consequently this missing energy may be ignored.

Having now defined the system matrix (Hamiltonian) H by eq. (11) for the electron interacting with the field, the evolution equation (2) for the three observables $\{\rho_j : j = 1, 2, 3\}$ may be determined, with the result that

$$
\begin{aligned}
\partial_t\rho_1 &= & \Omega_0\rho_2 & \\
\partial_t\rho_2 &= & -\Omega_0\rho_1 & - 2eq_0\hbar^{-1}E\rho_3 \\
\partial_t\rho_3 &= & 2eq_0\hbar^{-1}E\rho_2 &
\end{aligned}
\qquad (12)
$$

It is easily verified that $\rho_1^2 + \rho_2^2 + \rho_3^2 = 1$ again in this time-dependent system.

This system of ODEs can be used to study the Stark effect in atoms due to an applied *static* electric field $E = \mathrm{constant}$. With the rotating wave approximation (RWA) it can be used to study stimulated absorption, stimulated emission and Rabi oscillations due to an applied CW field $E = E_0\cos\Omega t$ with frequency Ω and constant amplitude E_0, as is done in many textbooks of quantum electronics.[1–3] The rotating wave approximation is introduced by the substitutions

$$
\begin{aligned}
\rho_1 &= \mathrm{Re}\{se^{-\mathrm{i}\Omega t}\} \\
\rho_2 &= \mathrm{Im}\{se^{-\mathrm{i}\Omega t}\} \\
E &= \mathrm{Re}\{E_0 e^{-\mathrm{i}\Omega t}\}
\end{aligned}
\qquad (13)
$$

with constant E_0 which may be complex. After neglecting second-harmonic terms (the rotating wave approximation) the system eq. (12) reduces to

$$\begin{aligned} i\partial_t s &= \Delta s - eq_0\hbar^{-1}E_0\rho_3 \\ \partial_t\rho_3 &= eq_0\hbar^{-1}\text{Im}\{E_0^* s\} \end{aligned}$$

(14)

where $\Delta = \Omega_0 - \Omega$ is the *detuning* of the carrier frequency of the electromagnetic field from the resonance of the quantum system.

One can also derive from eqs. (12), by eliminating $\partial_t\rho_2$, the Lorentz-like equation

$$\partial_t^2\rho_1 + \Omega_0^2\rho_1 = -2eq_0\Omega_0\hbar^{-1}E\rho_3,$$

(15)

which has the appearance of a forced harmonic oscillator with natural resonant frequency Ω_0. The presence of the term ρ_3 on the right hand side has a crucial effect on the behaviour of the forced system, since $\rho_3 = \{1 - (\rho_1^2 + \rho_2^2)\}^{1/2} = \{1 - (\rho_1^2 + \Omega_0^{-2}(\partial_t\rho_1^2))\}^{1/2}$. This relation renders the differential equation (15) nonlinear. However, for small excitations of the atom from the ground state for which $\rho_3 \sim 1$, it may be approximated by

$$\partial_t^2\rho_1 + \Omega_0^2\rho_1 \sim -2eq_0\Omega_0\hbar^{-1}E.$$

(16)

This is a simple forced harmonic oscillator with a linear response to the field E, exactly as predicted by the Lorentz model of the electron.[6]

5. COUPLING THE ATOM TO THE FIELD

In order to complete the description of the coupled atom-field sysytem it is necessary to represent the driving effect of the atomic dipole polarisation as a source for the electromagnetic field. Thus we define the *polarisation density P* by

$$P = -Neq = -Ne\langle Q\rangle = -Neq_0\rho_1$$

(17)

where N is the density of the atoms. Maxwell's equations for a 1-dimensional medium (plane waves propagating along the z-direction in a 3-dimensional medium) reduce to

$$\partial_z^2 E - c^{-2}\partial_t^2(E + \epsilon_0^{-1}P) = 0.$$

(18)

6. PROBABILITY INTERPRETATIONS OF QUANTUM MECHANICS

Many of the properties of expectations of operators are similar to the fundamentals of probability theory. Given an operator A and the quantum state-vector $v(t)$, the state may be expanded in a superposition of (orthonormal) eigenvectors $\{w^{(k)} : k = 1, \ldots, n\}$ of A

$$v(t) = \sum_{k=1}^n c_k(t)w^{(k)}$$

(19)

with complex scalar coefficients $c_k(t) = \langle w^{(k)}, v(t)\rangle$. In this representation, the expectation of the operator A is

$$\langle A\rangle = \sum_{k=1}^n a_k|c_k|^2$$

(20)

with a_k the k-th eigenvalue of A corresponding to the eigenvector $w^{(k)}$. Since the coefficients can be shown to satisfy $0 \le |c_k|^2 \le 1$, eq. (20) is similar to the expression for the statistical expectation of the discrete random variable A having values a_k with probability $P(A = a_k) = |c_k|^2$. The statistical interpretation of M. Born is that a measurement of A yields the value a_k with probability $|c_k|^2$, and the average value of such measurements is the expectation $\langle A\rangle$. When applied to the measurement of the energy of the system $\langle H\rangle$, we would say that the probability that the electron has energy W_k is $|v_k|^2$.

The probability interpretation of energy-level occupancy permits the interpretation of the third equation of eq. (12) or eq. (14) as a *rate equation*, describing the rate at which the level occupancies change subject to transitions induced by external fields such as stimulated absorption or emission. This interpretation can be qualitatively generalised to describe non-conservative processes such as energy pumping from an external source or nonradiative relaxations of the electron state, by adding to the quantum rate equation phenomenological terms which describe these transition rates. In particular, an additional term $-T^{-1}(\rho_3 - \rho_{30})$ added to the right hand side of these equations describes an external forcing of the level occupancy towards the constant value ρ_{30} with a characteristic time constant T. The choice $\rho_{30} = -1$ implies driving towards the upper state (pumping) and $\rho_{30} = +1$ towards the ground state (dissipative relaxation). Both these mechanisms are important in lasers. The rate equations can be substantially modified, for example to describe more complex pumping routes, without any further modifications of the quantum theory of the coherent polarisation.

These refinements essentially complete a quantum electronic theory which is capable of describing most kinds of laser action between two active quantum levels, while not requiring any of the apparatus of conventional wavefunction-based quantum mechanics.

7. QUANTUM THEORY OF THE 1-ELECTRON ATOM USING A WAVEFUNCTION

Although the theoretical framework developed here is adequate for the description of laser action in a resonant 2-level medium, it is very desirable to connect this theory to the more conventional quantum description of atoms, in which the state-vector is replaced by a continuous wavefunction. In order to do this, it is possible to make further use of the probability interpretation to introduce the *position representation* of quantum mechanics.

It would be very convenient to be able to formulate other probability-like statements, such as the probability (density) that the electron is at a certain position $\mathbf{x}$. This would require a state vector indexed by the *continuous* variable $\mathbf{x}$ rather than the *discrete* index so far considered for the state vector. This leads to the replacement of the discrete state vector, with elements v_k, by the continuous wavefunction $\psi(\mathbf{x})$, such that $|\psi(\mathbf{x})|^2$ is the required probability density function. Proper normalisation of the probability density function would require that $\int_{R^3} |\psi(\mathbf{x})|^2 \mathrm{d}^3\mathbf{x} = 1$. This in turn suggests the inner product $\langle \phi, \psi \rangle = \int_{R^3} \phi^*(\mathbf{x})\psi(\mathbf{x})\mathrm{d}^3\mathbf{x}$. It is easy to show how a discrete state vector can be turned into a continuous wavefunction, for example by $\psi(\mathbf{x}) = \sum_{k=1}^n v_k \psi_k(\mathbf{x})$ where the functions $\{\psi_k\}$ form an arbitrary orthonormal set under the inner product for wavefunctions. Many examples of such transformations can be easily demonstrated, for example elementary Fourier series with the space R^3 replaced by the interval $-\pi < x < \pi$ and $\psi_k(x) = (2\pi)^{-1/2}\exp(\mathrm{i}kx)$. In this sense, the wavefunction ψ and the state vector v contain precisely the same *information*, which is just expressed in different ways.

The definition of the wavefunction $\psi(\mathbf{x})$ in this way, along with the associated Hilbert space structure of inner products, leads to the *position representation* of quantum mechanics. In this representation, matrices are replaced by linear operators such as differential operators. One operator is privileged in this representation, namely the position operator $\mathbf{Q}$ whose expectation $\langle \mathbf{Q} \rangle$ yields the average value of a measurement of position in space of the electron. The operator $\mathbf{Q}$ is diagonal in position representation, which means that the eigenvalues of each Cartesian component operator Q_j ($j = 1, 2, 3$) are just the continuum of parameter values x_j, so that the action of this operator on the wavefunction is simply multiplication

$$\mathbf{Q}\psi(\mathbf{x}) = \mathbf{x}\psi(\mathbf{x}). \tag{21}$$

The expectation $\langle \mathbf{Q} \rangle$ in the state defined by the wavefunction $\psi(\mathbf{x})$ is therefore given directly by the equivalent of eq. (20) for operators,

$$\mathbf{q} = \langle \mathbf{Q} \rangle = \int_{R^3} \mathbf{x}|\psi(\mathbf{x})|^2 \mathrm{d}^3\mathbf{x}. \tag{22}$$

The total energy of a particle of charge $-e$ and mass m moving around a point charge $+e$ is give in classical mechanics by

$$W = \frac{1}{2m}\mathbf{p} \cdot \mathbf{p} - \frac{e^2}{4\pi\epsilon_0||\mathbf{q}||} \tag{23}$$

where $\mathbf{p}$ is the momentum of the particle, equal to the product of mass and velocity, and $\mathbf{q}$ is the position of the negative charge with respect to the positive point charge. We have an operator replacement $\mathbf{Q}$ for the position $\mathbf{q}$, which is trivial because it is diagonal in position representation. To fully quantise the classical energy and thereby translate the classical energy into an operator H, we also need an operator for the momentum $\mathbf{p}$, which is not so trivial to find. Once this operator is determined, the Hamiltonian operator H for the energy is

$$H = \frac{1}{2m}\mathbf{P}\cdot\mathbf{P} - \frac{e^2}{4\pi\epsilon_0||\mathbf{Q}||} \tag{24}$$

As is well-known, the required operator for the momentum is $\mathbf{P} = -i\hbar\boldsymbol{\nabla}$, but it is not easy to give an elementary motivation for this choice without introducing the Poisson bracket and its isomorphism with the commutator of operators, which is the standard method in quantum mechanics textbooks. A simple, though not completely convincing, argument is that

$$\langle\mathbf{P}\rangle = m\partial_t\langle\mathbf{Q}\rangle = (i\hbar)^{-1}m\langle[\mathbf{Q},H]\rangle \tag{25}$$

which suggests

$$\begin{aligned}\mathbf{P} &= (i\hbar)^{-1}m[\mathbf{Q},H] \\ &= (2i\hbar)^{-1}[\mathbf{Q},\mathbf{P}\cdot\mathbf{P}]\end{aligned} \tag{26}$$

because $\mathbf{Q}$ commutes the with the second term in H. One can check easily in 1-dimension with $Q = x$ that the operator P which satisfies this condition is $P = -i\hbar\partial_x$, because

$$\begin{aligned}[Q,\partial_x^2]\psi &= x\partial_x^2\psi - \partial_x^2(x\psi) \\ &= -2\partial_x\psi\end{aligned} \tag{27}$$

and a further check reveals that $\mathbf{P} = -i\hbar\boldsymbol{\nabla}$ is the correct generalisation for 3-dimensions. This derivation is actually independent of the form assumed for the position-dependent potential in the second term in the energy, which may even be removed for a free particle, with the same result for the momentum operator.

Having obtained a complete operator representation for the 1-electron atom, one can go on to describe standard textbook constructions of the theory of hydrogen atom energy levels, particle in a box and some semiconductor phenomena such as quantum wells. It remains to then show how the 2-state theory of the atom used at the introductory stages is recovered by the hypothesis that the wavefunction ψ is given by a superposition of the eigenfunctions of the energy operator

$$\psi(\mathbf{x},t) = \sum_k v_k(t)\psi_k(\mathbf{x}). \tag{28}$$

For interactions with external fields which are nearly resonant with the two lowest energy states $k = 1$ and $k = 2$, the wavefunction may be approximated by restricting the sum to the two lowest states

$$\psi(\mathbf{x},t) = v_1(t)\psi_1(\mathbf{x}) + v_2(t)\psi_2(\mathbf{x}), \tag{29}$$

where the complex coefficients $\{v_1, v_2\}$ form the two elements of the state vector.

8. CONCLUSIONS

The method of constructing quantum electronic theory from matrix mechanics, using only systems of ordinary differential equation, is advocated as a means of introducing the subject to undergraduates in Engineering, who are generally conversant with the mathematics of circuit theory but not with that of Hamiltonian mechanics, partial differential equations and separation of variables as required by most conventional treatments of the subject.

Despite the relative simplicity and accessibility of the mathematics employed in this treatment, it should be stated that there are undoubted difficulties present as well. The theory may appear quite formal and abstract, with very little to connect it to a reality that can be visualised by a student, particularly in the early stages of the development. Since the approach eschews the wave pictures, Böhr model, and wavefunction localisation of the conventional position-representation method, alternative motivations have to be found for students to believe that the model can make physical sense, and is not just a game played with mathematics. Nevertheless, the advantages of relative mathematical simplicity and relevance to the resonant interactions of lasers are very substantial, and make the subject accessible from a viewpoint which is familiar to students who can appreciate circuit theory and control system theory.

9. ACKNOWLEDGEMENTS

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