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Mahmud Sakah
Brahim Chebbi
Ilya Golub

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Mahmud Sakah,[a] Brahim Chebbi,[a,*] and Ilya Golub[b]
[a]Laurentian University, Bharti School of Engineering, Ramsey Lake Road, Sudbury, Ontario P3E 2C6, Canada
[b]Algonquin College, School of Advanced Technology, Woodroffe Avenue, Ottawa, Ontario K2G 1V8, Canada

Abstract. We analyze theoretically, numerically, and experimentally the spectral response of scattered light intensity from moving particles crossing the fringes of a Bessel beam. This response could be the basis of a simple technique to measure velocity. © *The Authors. Published by SPIE under a Creative Commons Attribution 3.0 Unported License. Distribution or reproduction of this work in whole or in part requires full attribution of the original publication, including its DOI.* [DOI: 10.1117/1.OE.54.8.084106]

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Laser Doppler velocimetry (LDV) is a very well established technique for measuring fluid and solid surface velocities. The most widely used configuration of this technique is the dual beam mode, in which two beams intersect at the measuring volume and the modulation frequency of scattered light is given by the difference of the Doppler frequencies. This frequency difference depends on the angle of intersection and is independent of the point of observation. The fringe model, frequently used to describe this relationship, relates the Doppler frequency difference to the fringe spacing of the two intersecting beams.[1] Usually, refractive and reflective elements (lenses, mirrors, and beam splitters) are used to intersect the beams, however, diffractive holographic elements were also suggested and proved to produce a fringe pattern that is stable against wavelength drift and laser mode hoping.[2] Also, methods for the formation of fringes other than using the intersection of two Gaussian beams were suggested in the literature. For example, the interference of three coplanar Gaussian beams was used to form the measuring volume and measure the particle position and microflow velocity profile with increased spatial resolution.[3] In another investigation, two nearly Bessel beams were intersected to form the measuring volume of an LDV system.[4] A technique related to the LDV, called laser Doppler flowmetry (LDF), is used for estimation of blood flow and perfusion measurements on skin and organs. In this technique, a single Gaussian beam is used.

In the present investigation, we explore the use of the fringes of one Bessel beam to measure velocity. The concept we propose is shown in Fig. 1. We measure the intensity of light scattered from moving particles crossing the fringes of a Bessel beam created by an axicon. The frequency content of the obtained signal is analyzed for information about the velocity of the particles. The Bessel beam fringes are axisymmetric and their intensity is not uniform. In most cases, using parallel fringes with uniform spacing is better than using Bessel beam fringes. The technique suggested in the present research is, in its current form, less advantageous than regular LDV in terms of accuracy, resolving velocity components, flow direction, and longitudinal spatial resolution. However, this method has a simple and compact configuration and can offer an advantageous alternative in some applications, for example, in solid surface measurements or in one-dimensional microfluidic flows, where compactness is important. More importantly, this technique might be useful for measurements in turbid media such as in LDF. Bessel beams have distinct properties, namely near diffractionless, long depth-of-field (DOF), and self-healing. Because of their ability to reconstruct after being disturbed by an obstacle, they are particularly advantageous for use in turbid media for which they were used in applications such as micromanipulation[5] and microscopy.[6] Moreover, one of the main limitations of LDF is the depth in tissue where measurements are performed. There is no control on the depth of measurement and the most widely used method to measure at different depths is placing the detector at varying distances from the source to pick up scattered light from different regions.[7] Bessel beams offer an opportunity to reach larger depth and to control the depth measurements more efficiently. It is possible to control the longitudinal intensity distribution of Bessel beams by using logarithmic,[8] linear, and exponential axicons[9] and to compensate for the losses in turbid media.[10] Finally, the study of the spectral response of light scattered from particles crossing the fringes of a Bessel beam is important on its own merit and can be useful in other applications where Bessel beams are used, such as in micromanipulation and microscopy.

The present concept is different from the one used in the system reported in Ref. 4, which is an LDV system using two Bessel beams to create a measurement volume with parallel fringes. It should be mentioned that another investigation reported the use of Bessel beams in ultrasound Doppler velocity estimation.[11] Because the field is measured in an ultrasound system, it has a different spectral response than the current optical system, where the intensity is measured.

We perform a theoretical analysis by considering a particle crossing the fringes of a Bessel beam with a constant velocity normal to the longitudinal axis and passing through the center of the beam. For simplicity, we use a very basic model for the light scattered from this particle and collected by the detector. We consider that the particle size is smaller than the fringes, and we assume that the intensity of this light is proportional to the intensity of the fringes it crosses. It is well established that from an incoming Gaussian beam, a regular axicon of base angle α produces an intensity proportional to $J_0^2(k\beta r)$, where J_0 is the Bessel function of the first kind of order 0, r is the radius from the center of the beam, k is the wave number, and β is the refraction angle. The wave number is given by $k = 2\pi/\lambda$, and the refraction angle is approximated for small angles in terms of α by $\beta \approx (n-1)\alpha$, where λ is the wavelength and n is the index of refraction of the axicon. Thus, as the particle crosses the fringes through the center, it scatters light with intensity proportional to $J_0^2(k\beta r)$ and a photodetector collecting this light produces a proportional electrical signal (particle 1 in Fig. 1). If the

*Address all correspondence to: Brahim Chebbi, E-mail: bchebbi@laurentian.ca

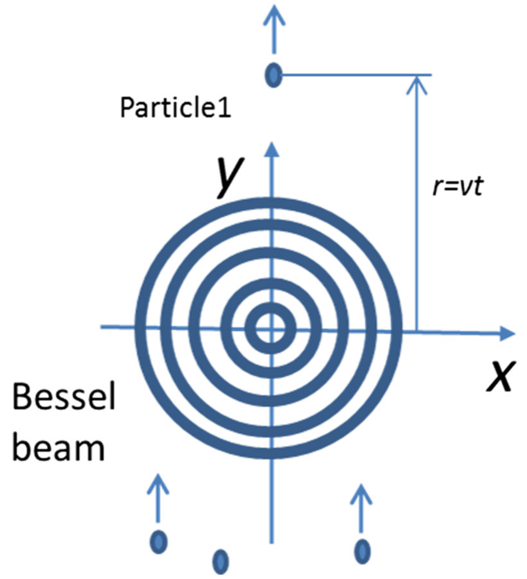

Fig. 1 Scattering of light from particles crossing Bessel beam fringes.

particle is moving with a velocity v parallel to the y axis, we can substitute $r = vt$ and the intensity as a function of time, t, is proportional to $J_0^2(k\beta vt)$. The spectrum, $F(f)$, as a function of the frequency f(Hz), of this signal is obtained by calculating its Fourier transform, $\mathcal{F}$, which is given using the distribution theory.[12]

$$F(f) = \mathcal{F}([J_0(k\beta vt)]^2) = \begin{cases} \frac{2}{f_b \pi^3} K\left(1 - \frac{f^2}{f_b^2}\right), & f < f_b \\ 0, & \text{otherwise} \end{cases}. \quad (1)$$

$K()$ is the complete elliptic integral[13] of the first kind given for a parameter m by $K(m) = \int_0^1 \mathrm{d}t/\sqrt{(1-t^2)(1-mt^2)}$ and $f_b = k\beta v/\pi$, which can be written as

$$f_b = \frac{2\beta v}{\lambda}. \quad (2)$$

$F(f)$ is plotted if Fig. 2 for $\alpha = 5$ deg, $\lambda = 658$ nm, and $n = 1.52$ and different velocities. It has a half-hut shape, high at low frequencies, and decreases until f_b, after which it becomes zero. The expression of the frequency, f_b, which is called the Bessel frequency,[4] is similar to the expression of the Doppler frequency shift, f_D, for the LDV dual beam mode, where β is the half angle between the two intersecting beams. The difference is that in the case of the LDV, the spectrum shows a peak at f_D, while in this case f_b marks the edge of the hut-shaped frequency distribution.

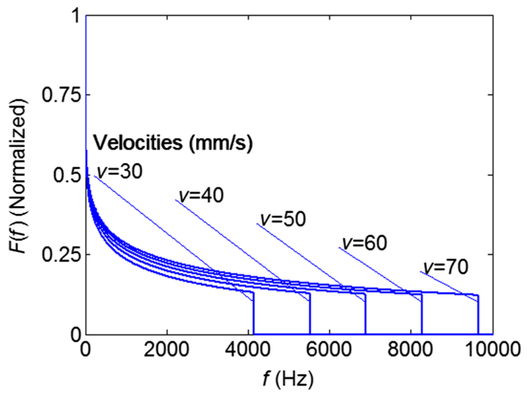

Fig. 2 Fourier transform of $J_0^2(k\beta vt)$.

This analysis considers only the simplest case where a particle crosses the Bessel beam at its center. Numerical simulations were performed to deal with the more general cases where the particles can cross the beam at other trajectories with respect to the center. A time signal of scattered light intensity was generated by allowing one particle to move at constant velocity in the y direction from any starting point in the (x, y) plane, as shown in Fig. 1. The intensity is assumed to be proportional to that of the fringes where the particle is located as it moves, and the spectral response of the generated signal was numerically calculated by using the fast Fourier transform. The results are plotted in Fig. 3 for particles crossing the Bessel beam at different cross-sections for a velocity of 50 mm/s. For a particle crossing the center ($x = 0$ mm, $y = -1$ mm), the results are consistent with the theoretical results in Fig. 2.

The spectra for particles crossing off-center have the same value of the limiting frequency, f_b, with oscillation at lower frequencies depending on the trajectory of the particle. As expected, the signal strength decreases as the trajectories get away from the center. Since f_b is the quantity, which relates to velocity, these curves can be used to infer the velocity value.

Experiments were conducted to validate the theoretical and numerical simulations. A red laser beam ($\lambda = 658$ nm) was expanded to a diameter of ~8 mm and passed through a 5 deg axicon to generate a nearly Bessel beam with a DOF ~86-mm long and an average fringe spacing of ~7 μm. A sandpaper sheet was mounted on a linear actuator and allowed to move at known different constant velocities with its surface in the plane (x, y) of Fig. 1 perpendicular to the direction of propagation of the Bessel beam. The scattered light from particles of the sandpaper crossing the fringes of the Bessel beam was focused by a lens on the photodetector. In order to obtain a steady state and constant velocity during a reasonable time period, the sandpaper was selected to be large compared to the size of the Bessel beam. The signal from the photodetector was amplified, lowpass filtered, digitized, and processed. The spectrum of the signal was computed and the results are shown in Fig. 4 for different velocities. Polynomial curve fitting for the spectra was used and is shown in the figure. The theoretical values of the Bessel frequency calculated using Eq. (2) are also shown. In general, the experimental results agree with the theoretical analysis and numerical simulations in terms of the dependence of the spectra on the velocity. The main differences, which are mostly due to the finite size of the particles crossing the Bessel beam, are that the experimental spectrum of the scattered light shows variations and it is not well defined at the Bessel frequency. For the theoretical and numerical simulation, the spectrum falls off abruptly to a zero value at f_b while experimentally it does so gradually. Nevertheless, the value at which the decrease starts corresponds well with the theoretical value. Equation (2) can then be used to calculate the velocity from the measured spectrum.

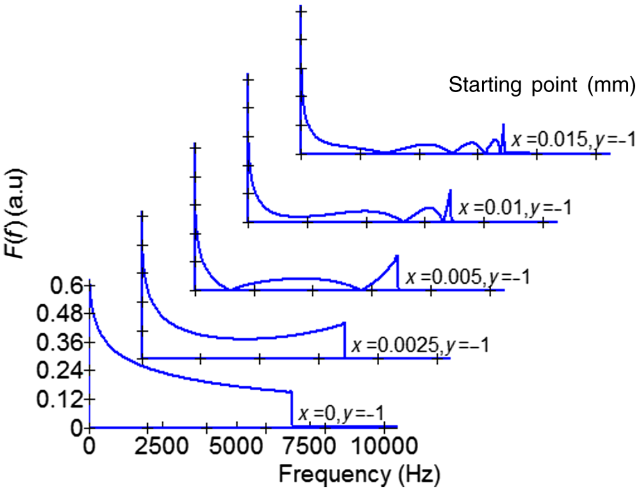

Fig. 3 Numerical simulation of scattered light spectra from particles crossing Bessel beam at different positions at a speed $v = 50$ mm/s.

In conclusion, the results presented can be the basis for the development of a new technique to measure velocity. We call this technique the laser Bessel velocimetry, since even though it resembles LDV, it has a completely different spectral response specific to Bessel beams. This technique, as presented, does not have a confined measuring volume and does not give velocity direction or components. These limitations can be further investigated, for example, one can use the receiving optics to focus on the desired measuring location or detectors placed at different positions to resolve the components. Also, there are ways of limiting the extent of the measuring volume in the longitudinal direction and remotely controlling its location by using a three axicons system similar to the system presented in Ref. 14. Finally, this technique has more potential for use in turbid media, for example, to measure blood perfusion where flow direction is less important and where the self-healing properties of Bessel beams and the ability to control their longitudinal intensity distribution are advantageous.

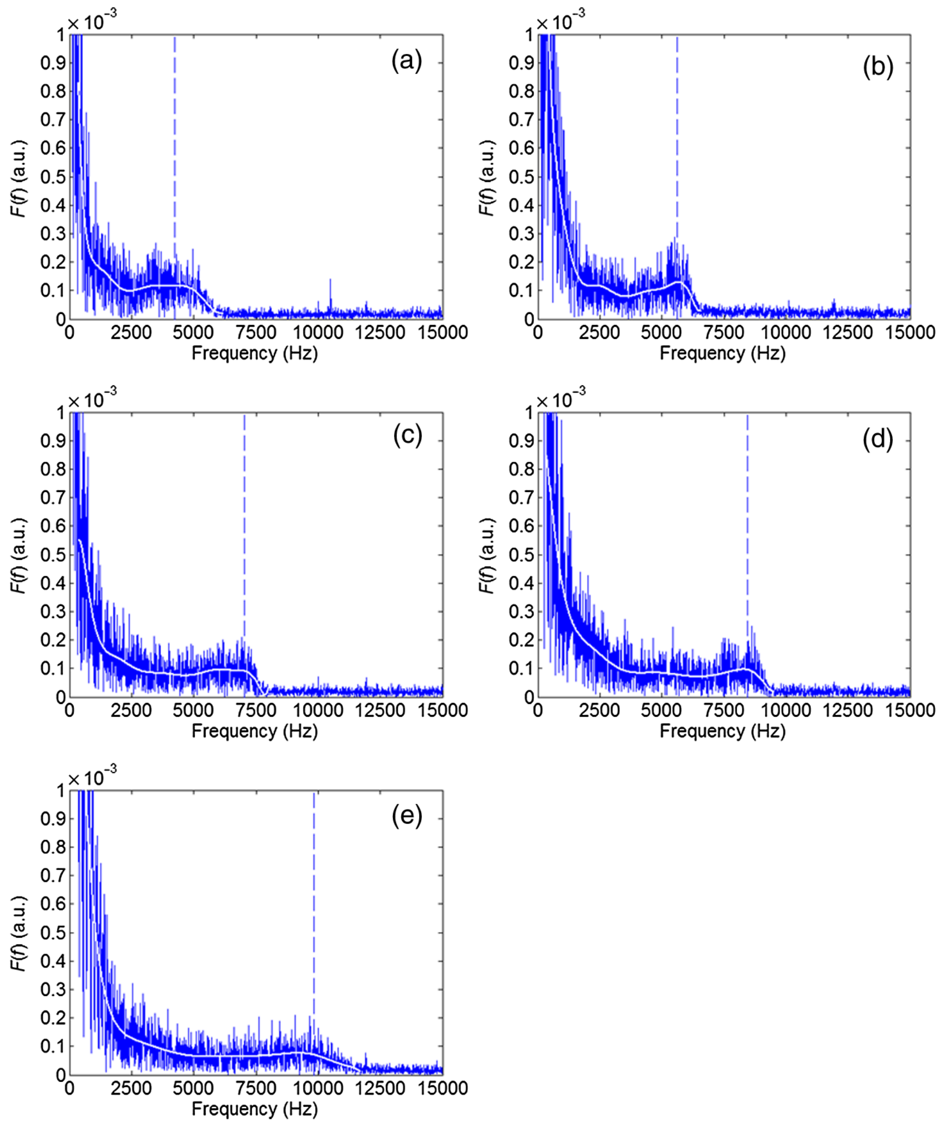

Fig. 4 Experimental results of Fourier transform of intensity. Dotted line shows theoretical value [Eq. (2)]. White lines show polynomial curve fitting. (a) $v = 30$ mm/s. (b) $v = 40$ mm/s. (c) $v = 50$ mm/s. (d) $v = 60$ mm/s. (e) $v = 70$ mm/s.

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