Rayleigh or Abbe? Origin and naming of the resolution formula of microlithography

Anthony Yen*
ASML Technology Development Center, San Jose, California, United States

Abstract. We review the history in connection with the resolution formula of microlithography and argue that it was Abbe rather than Rayleigh who definitively stated the $0.5\frac{\lambda}{\mathrm{NA}}$ resolution limit for the minimum pitch first, using an approach more relevant to projection imaging, and hence, this expression should be more appropriately referred to as the Abbe formula for the resolution of a projection imaging system. © *2020 Society of Photo-Optical Instrumentation Engineers (SPIE)* [DOI: 10.1117/1.JMM.19.4.040501]

Keywords: Ernst Abbe; microlithography; minimum pitch; Rayleigh's equation; resolution limit; k_1.

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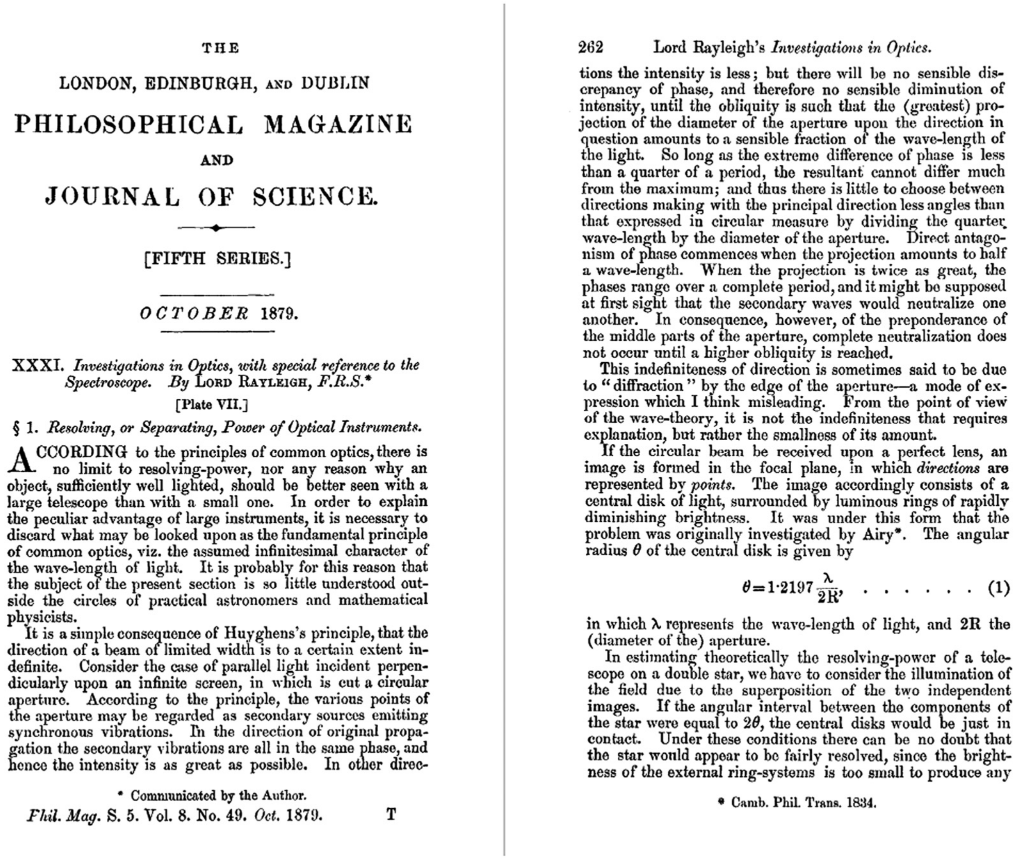
Every semiconductor lithographer seems to be aware that the resolution of a projection-imaging lithographic system can be described by what is commonly called Rayleigh's equation, which says that the printable minimum half-pitch in the photoresist is $k_1\frac{\lambda}{\mathrm{NA}}$ where λ is the exposing wavelength, NA is the numerical aperture of the projection optics, and k_1 depends on several factors such as the configuration of the illuminator and the resolution of the photoresist in which a relief image of the pattern on the photomask is printed (the earliest reference to this name that we could find is Ref. 1, in which the formula here and the one for the depth of focus were referred to as the Rayleigh criteria.). He or she also knows that k_1 has a lower bound of 0.25. In this letter, we argue that it was Abbe who definitively stated the $0.5\frac{\lambda}{\mathrm{NA}}$ resolution limit (for pitch instead of half-pitch) first, using an approach more relevant to projection imaging, and hence, the above expression should be more appropriately referred to as the Abbe formula for the resolution of a projection imaging system. (To be clear, this is not the first time the name "Abbe formula" is mentioned. Others have already used this name in their various publications.)

The Rayleigh criterion for resolution originates from Lord Rayleigh's 1879 article[2] (see Fig. 1), though Helmholtz had already come up with the $0.5\frac{\lambda}{\mathrm{NA}}$ resolution limit using similar arguments in 1874 (see caption of Fig. 5; this fact was also acknowedged by Rayleigh in a later article of his). In the beginning part of this article, he put forward the formula obtained by Airy in 1834,

$$\theta = 1.2197\frac{\lambda}{2R},$$

where θ is the angular radius of the bright central disk, λ represents the wavelength of the light, and $2R$ is the diameter of the circular aperture in front of a perfect lens, and went on to state that "in estimating theoretically the resolving-power of a telescope on a double star, we have to consider the illumination of the field due to the superposition of the two independent images. If the angular interval between the components of the star were equal to 2θ, the central disks would be just in contact. Under these conditions there can be no doubt that the star would appear to be fairly resolved, since the brightness of the external ring-systems is too small to produce any material confusion, unless indeed the components are of very unequal magnitude." He then went on to discuss two neighboring luminous lines and proposed his resolution criterion that is more lenient than above. Such luminous lines were generated in prism or grating spectroscopes by

*Address all correspondence to Anthony Yen, tony.yen@asml.com



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[FIFTH SERIES.]

OCTOBER 1879.

XXXI. *Investigations in Optics, with special reference to the Spectroscope.* *By* LORD RAYLEIGH, *F.R.S.**

[Plate VII.]

§ 1. *Resolving, or Separating, Power of Optical Instruments.*

ACCORDING to the principles of common optics, there is no limit to resolving-power, nor any reason why an object, sufficiently well lighted, should be better seen with a large telescope than with a small one. In order to explain the peculiar advantage of large instruments, it is necessary to discard what may be looked upon as the fundamental principle of common optics, viz. the assumed infinitesimal character of the wave-length of light. It is probably for this reason that the subject of the present section is so little understood outside the circles of practical astronomers and mathematical physicists.

It is a simple consequence of Huyghens's principle, that the direction of a beam of limited width is to a certain extent indefinite. Consider the case of parallel light incident perpendicularly upon an infinite screen, in which is cut a circular aperture. According to the principle, the various points of the aperture may be regarded as secondary sources emitting synchronous vibrations. In the direction of original propagation the secondary vibrations are all in the same phase, and hence the intensity is as great as possible. In other direc-

* Communicated by the Author.

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tions the intensity is less; but there will be no sensible discrepancy of phase, and therefore no sensible diminution of intensity, until the obliquity is such that the (greatest) projection of the diameter of the aperture upon the direction in question amounts to a sensible fraction of the wave-length of the light. So long as the extreme difference of phase is less than a quarter of a period, the resultant cannot differ much from the maximum; and thus there is little to choose between directions making with the principal direction less angles than that expressed in circular measure by dividing the quarter wave-length by the diameter of the aperture. Direct antagonism of phase commences when the projection amounts to half a wave-length. When the projection is twice as great, the phases range over a complete period, and it might be supposed at first sight that the secondary waves would neutralize one another. In consequence, however, of the preponderance of the middle parts of the aperture, complete neutralization does not occur until a higher obliquity is reached.

This indefiniteness of direction is sometimes said to be due to "diffraction" by the edge of the aperture—a mode of expression which I think misleading. From the point of view of the wave-theory, it is not the indefiniteness that requires explanation, but rather the smallness of its amount.

If the circular beam be received upon a perfect lens, an image is formed in the focal plane, in which *directions* are represented by *points*. The image accordingly consists of a central disk of light, surrounded by luminous rings of rapidly diminishing brightness. It was under this form that the problem was originally investigated by Airy*. The angular radius θ of the central disk is given by

$$\theta = 1{\cdot}2197\frac{\lambda}{2R}, \quad \ldots \ldots \quad (1)$$

in which λ represents the wave-length of light, and 2R the (diameter of the) aperture.

In estimating theoretically the resolving-power of a telescope on a double star, we have to consider the illumination of the field due to the superposition of the two independent images. If the angular interval between the components of the star were equal to 2θ, the central disks would be just in contact. Under these conditions there can be no doubt that the star would appear to be fairly resolved, since the brightness of the external ring-systems is too small to produce any

* Camb. Phil. Trans. 1834.

Fig. 1 Lord Rayleigh's 1879 article on the resolution of two neighboring features.

light sources with two spectral lines very close in wavelength. Rayleigh first stated, quoting Airy and Verdet, that the intensity (which he called brightness) of a luminous spectral line was proportional to the square of the sinc function

$$\left(\frac{\sin\frac{\pi a\xi}{\lambda f}}{\frac{\pi a\xi}{\lambda f}}\right)^2,$$

where ξ is the horizontal axis, a is the horizontal dimension of the rectangular aperture (placed after the prism but before the focusing lens), and f is the focal length of the lens. He then tabulated the values of the above function and pronounced that if the two neighboring lines were so separated that the maximum intensity of one line fell onto the first minimum of that of the other line, then the two lines could be discerned, because the combined brightness in the middle of the two peaks (which have the brightness of 1) dipped down to 0.8106 (see Fig. 2). Hence, the smallest discernable separation d of the two lines was

$$d = \frac{\lambda}{a/f}.$$

If we translate this to our language, a/f is twice the NA of a one-dimensional lens in air. Hence, the Rayleigh criterion simply implies that the discernable separation of two neighboring lines is $0.5\frac{\lambda}{\mathrm{NA}}$. The same criterion can also be applied to the Airy patterns. If we allow the maximum of the first Airy pattern to coincide with the edge of the bright central disk of the second pattern, then the light intensity at the saddle point in the middle of the two intensity peaks is 0.7348 times the intensity at either peak, and the minimum discernable distance in this case is $0.61\frac{\lambda}{NA}$, as has been stated in many textbooks.

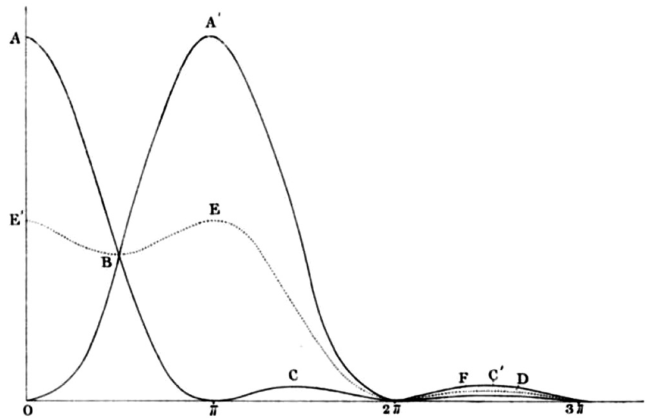


Fig. 2 Rayleigh's plots in his 1879 article. ABCD is $(\frac{\sin u}{u})^2$; OA′C′ is $(\frac{\sin(u-\pi)}{u-\pi})^2$; and E′BEF is half of $[(\frac{\sin u}{u})^2 + (\frac{\sin(u-\pi)}{u-\pi})^2]$.

What Rayleigh stated in his article can be easily explained. Light disturbance in the image plane, produced by a distant star, is simply the point-spread function of the optical system of the telescope, since the distant star can be regarded as a δ-function object. One can look up, in a number of textbooks (see e.g., Ref. 3, pp. 76–79), to find that the light intensity of the Airy disk which is the square of the point-spread function (the Fraunhofer diffraction of a circular aperture) is proportional to

$$\left[\frac{J_1\left(\frac{2\pi}{\lambda}\mathrm{NA}\cdot r\right)}{\frac{2\pi}{\lambda}\mathrm{NA}\cdot r}\right]^2,$$

where J_1 is the Bessel function of the first kind, order 1, whose first zero occurs at the argument of 1.22π, and NA, the numerical aperture of the optical system, equals Rayleigh's R/d with R being the radius of the aperture and d the distance from the aperture to the image plane. Setting the argument of the above Bessel function to 1.22π, the diameter of the Airy disk is then

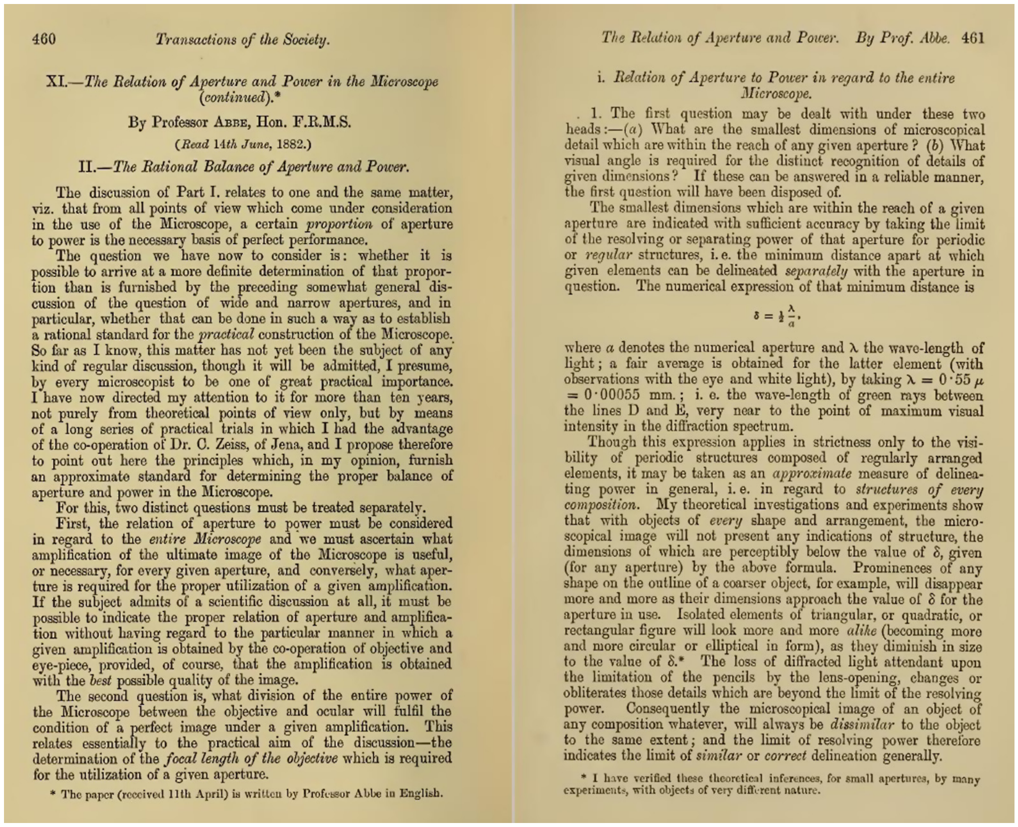
$$2r = 1.22\frac{\lambda}{\mathrm{NA}},$$

or as Rayleigh stated, the angular radius is

$$\theta = \frac{r}{d} = 1.22\frac{\lambda}{2R}.$$

The Fraunhofer diffraction of a rectangular aperture can also be easily evaluated to be the product of two sinc functions in the horizontal and the vertical directions, respectively (see e.g., Ref. 3, p. 76). If the vertical dimension of the aperture is much smaller than its horizontal dimension, then the intensity of the diffraction pattern close to the horizontal axis is the square of the sinc function given by Rayleigh.

Rayleigh understood, however, what he put forward was not the absolute resolution limit. He stated in the article that "this rule is convenient on account of its simplicity." Born and Wolf[4] also stated in their book that "no special physical significance is to be attached to the Rayleigh criterion, and from time to time other criteria of resolution have been proposed." Rayleigh dealt with incoherent illumination. Under incoherent illumination, light intensity of the final image is the sum of the intensities produced by each point or line alone (see e.g., Ref. 3, p. 135). For two neighboring lines, we may argue that their minimum discernable separation can even be $0.45\frac{\lambda}{\mathrm{NA}}$, as the intensity in the mid-point between the two peaks dips down to 0.954 times the intensity at either peak. To be extreme, one can even argue that a one-percent intensity dip at the mid-point should be considered discernment of the two features. In fact, more than a century ago, Sparrow[5] stated that he was able to discern the two lines, by direct vision and in positive and negative film,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XI.—*The Relation of Aperture and Power in the Microscope* (*continued*).*

By Professor Abbe, Hon. F.R.M.S.

(*Read 14th June*, 1882.)

II.—*The Rational Balance of Aperture and Power.*

The discussion of Part I. relates to one and the same matter, viz. that from all points of view which come under consideration in the use of the Microscope, a certain *proportion* of aperture to power is the necessary basis of perfect performance.

The question we have now to consider is: whether it is possible to arrive at a more definite determination of that proportion than is furnished by the preceding somewhat general discussion of the question of wide and narrow apertures, and in particular, whether that can be done in such a way as to establish a rational standard for the *practical* construction of the Microscope. So far as I know, this matter has not yet been the subject of any kind of regular discussion, though it will be admitted, I presume, by every microscopist to be one of great practical importance. I have now directed my attention to it for more than ten years, not purely from theoretical points of view only, but by means of a long series of practical trials in which I had the advantage of the co-operation of Dr. C. Zeiss, of Jena, and I propose therefore to point out here the principles which, in my opinion, furnish an approximate standard for determining the proper balance of aperture and power in the Microscope.

For this, two distinct questions must be treated separately.

First, the relation of aperture to power must be considered in regard to the *entire Microscope* and we must ascertain what amplification of the ultimate image of the Microscope is useful, or necessary, for every given aperture, and conversely, what aperture is required for the proper utilization of a given amplification. If the subject admits of a scientific discussion at all, it must be possible to indicate the proper relation of aperture and amplification without having regard to the particular manner in which a given amplification is obtained by the co-operation of objective and eye-piece, provided, of course, that the amplification is obtained with the *best* possible quality of the image.

The second question is, what division of the entire power of the Microscope between the objective and ocular will fulfil the condition of a perfect image under a given amplification. This relates essentially to the practical aim of the discussion—the determination of the *focal length of the objective* which is required for the utilization of a given aperture.

* The paper (received 11th April) is written by Professor Abbe in English.

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i. *Relation of Aperture to Power in regard to the entire Microscope.*

1. The first question may be dealt with under these two heads:—(*a*) What are the smallest dimensions of microscopical detail which are within the reach of any given aperture? (*b*) What visual angle is required for the distinct recognition of details of given dimensions? If these can be answered in a reliable manner, the first question will have been disposed of.

The smallest dimensions which are within the reach of a given aperture are indicated with sufficient accuracy by taking the limit of the resolving or separating power of that aperture for periodic or *regular* structures, i.e. the minimum distance apart at which given elements can be delineated *separately* with the aperture in question. The numerical expression of that minimum distance is

$$\delta = \tfrac{1}{2}\frac{\lambda}{a},$$

where a denotes the numerical aperture and λ the wave-length of light; a fair average is obtained for the latter element (with observations with the eye and white light), by taking $\lambda = 0 \cdot 55\,\mu = 0 \cdot 00055$ mm.; i. e. the wave-length of green rays between the lines D and E, very near to the point of maximum visual intensity in the diffraction spectrum.

Though this expression applies in strictness only to the visibility of periodic structures composed of regularly arranged elements, it may be taken as an *approximate* measure of delineating power in general, i. e. in regard to *structures of every composition*. My theoretical investigations and experiments show that with objects of *every* shape and arrangement, the microscopical image will not present any indications of structure, the dimensions of which are perceptibly below the value of δ, given (for any aperture) by the above formula. Prominences of any shape on the outline of a coarser object, for example, will disappear more and more as their dimensions approach the value of δ for the aperture in use. Isolated elements of triangular, or quadratic, or rectangular figure will look more and more *alike* (becoming more and more circular or elliptical in form), as they diminish in size to the value of δ.* The loss of diffracted light attendant upon the limitation of the pencils by the lens-opening, changes or obliterates those details which are beyond the limit of the resolving power. Consequently the microscopical image of an object of any composition whatever, will always be *dissimilar* to the object to the same extent; and the limit of resolving power therefore indicates the limit of *similar* or *correct* delineation generally.

* I have verified these theoretical inferences, for small apertures, by many experiments, with objects of very different nature.

Fig. 3 Abbe's second of the two 1882 articles appearing in the *Journal of the Royal Microscopical Society*. The resolvable minimum distance (i.e., the pitch) of a periodic structure is clearly indicated as $\delta = \frac{1}{2}\frac{\lambda}{a}$ where λ is the wavelength of the illuminating light and a is the NA of the system.

all the way down to where the second derivative of the combined intensity curve at point B in Fig. 2 reached zero, meaning no intensity dip at all, at 0.83 times the Rayleigh separation. Therefore, a criterion based on a two-point or two-line structure is ambiguous. Also, the locations of the two peaks in the image intensity for the $0.45\frac{\lambda}{\mathrm{NA}}$ case are not $0.45\frac{\lambda}{\mathrm{NA}}$ but $0.365\frac{\lambda}{\mathrm{NA}}$ from each other. The root cause of all this ambiguity lies in the continuous nature of the spatial frequencies of a two-point (or two-line) object; they are not sharp peaks (δ-functions or near δ-functions in the spatial frequency domain) associated with periodic or regular structures which are either passed or eliminated without ambiguity by the pupil aperture. Abbe investigated exactly such periodic or regular structures.

In April of 1882, Abbe[6] submitted a paper, written in English, to the Royal Microscopical Society (see Fig. 3), in which he stated that, for periodic and regular features, "the minimum distance apart at which given elements can be delineated separately with the feature in question" was

$$\delta = \frac{1}{2}\frac{\lambda}{a},$$

where λ is the wavelength of the illuminating light and a is the NA of the system. We are not sure whether Abbe knew of Rayleigh's prior publication. Even if he knew, he made no reference to that work in this paper. Instead, Abbe stated that he had worked on this topic "for more than ten years," both theoretically and experimentally. Indeed, the first paper published by Abbe on the theory of microscopes appeared in 1873.[7] It is fifty-three pages of pure text without a single equation. One of the most succinct remarks in that paper is shown in Fig. 4.

One might argue that such a verbal description is no substitution for a derivation of the formula. True, while Abbe stated in 1882 that he had worked on the subject both theoretically and experimentally for more than ten years, he did not publish a derivation of his resolution formula

19. In Anschluss an die vorstehenden, für den wissenschaftlichen Gebrauch des Mikroskops wichtigen Schlussfolgerungen ergeben sich ferner ganz bestimmte Grenzen für das Unterscheidungsvermögen sowohl jedes einzelnen Objectivs, wie auch des Mikroskops überhaupt.

Durch kein Mikroskop können Theile getrennt (oder die Merkmale einer real vorhandenen Structur wahrgenommen) werden, wenn dieselben einander so nahe stehen, dass auch der erste durch Beugung erzeugte Lichtbüschel nicht mehr gleichzeitig mit dem ungebeugten Lichtkegel in das Objectiv eintreten kann. Daraus entspringt für jede Grösse des Oeffnungswinkels eine bestimmte kleinste Distanz des Unterscheidbaren, deren numerische Angabe nur desshalb un-

Fig. 4 Part of page 455 of the journal that contains Abbe's 1873 article. The emphasized words (printed in oversized letters) read: "Parts can be separated (or features of an actually existing structure can be perceived) by no microscope, if they are situated so close to each other that the first light bundle created by diffraction cannot enter the objective along with the undiffracted light cone."

during his lifetime. However, an extended version of Abbe's mathematically oriented lecture notes was later published by Lummer, who attended Abbe's lectures in 1887, and Reiche after Abbe's death.[8] (An English translation of this book, to be published by SPIE Press, is in preparation.) In it, the $\frac{1}{2}\frac{\lambda}{\mathrm{NA}}$ formula was given (see Fig. 5) in connection with an extensive write-up on the limit of resolution of imaging. The book and the remark on the timing of Helmholtz's work (also shown in Fig. 5) is the best evidence there is to substantiate the argument that Abbe had a full understanding of this subject and had arrived at his resolution formula by 1873.

A simple derivation of the Abbe resolution formula may proceed as follows. If we illuminate the object which is a grating of pitch p, the directions of diffracted beams obey the following grating equation (for a simple derivation of the grating equation, see e.g. Ref. 3, pp. 463–464)

$$n \sin \theta_m - n \sin \theta_i = m \frac{\lambda}{p},$$

Dieser niedrigste Grad der Ähnlichkeit wird bei zentraler Beleuchtung erreicht für:

$$\gamma = \frac{\lambda_0}{A} \qquad (80)$$

wobei außer dem Hauptmaximum noch beide benachbarten Maxima zur Wirksamkeit gelangen. Derselbe niedrigste Grad der Ähnlichkeit wird aber auch schon erreicht, wenn außer dem Hauptmaximum nur eins der beiden benachbarten Maxima mitwirkt. Dies kann verwirklicht werden durch Anwendung schiefer Beleuchtung, wobei die Gitterkonstante abnehmen darf bis auf den Wert:

$$\gamma_m = \frac{\lambda_0}{2A} \qquad (81)$$

Mit diesem Werte ist die Grenze der Auflösungsfähigkeit eines mikroskopischen Systems erreicht.

Bekanntlich kam Helmholtz[1]) fast zu gleicher Zeit, wenn auch auf anderem Wege, zur gleichen Grenze der Leistungsfähigkeit.

Geht man unter Benutzung der vollen Apertur $A = n$ vom Gitter

$$\gamma < \frac{\lambda_0}{2A}$$

[1]) H. Helmholtz: „Die theoretische Grenze für die Leistungsfähigkeit der Mikroskope". Pogg. Ann., Jubelband 1874, S. 557—584; Wissenschaftl. Abhandl. Bd. II, S. 185—212, 1883.

Fig. 5 Part of page 95 of Ref. 8 describing the resolution limit of the microscope. Here γ is the pitch of the grating, λ_0 is the wavelength of the incident light, and A is the NA of the system. It states that with the pitch in Eq. 81, the lowest degree of similarity (to the object) is obtained by combining the zeroth-order and one of the first-order diffractions, using oblique illumination. It also points out that Helmholtz came up with the same resolution limit in another way almost at the same time.

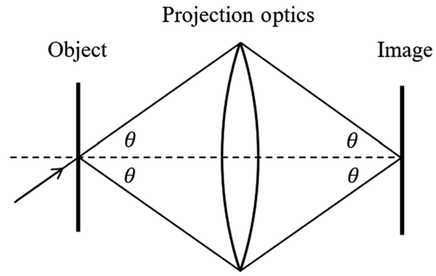

Fig. 6 Configuration employed to achieve maximum resolution of the object.

where m is the order of the diffracted beam, n is the index of refraction of the medium, θ_i is the angle of the incident beam, and θ_m is the angle of the m'th order diffracted beam. Following Abbe, to form an image, at least two of the diffracted beams from the object have to be collected and brought to interfere by the imaging optics. The pitch of the standing wave as a result of the interference of these two beams is

$$p' = \frac{\lambda}{2n \sin\theta},$$

where θ is half the angle formed by the two interfering beams. This formula is easily obtained since the normalized intensity of the standing wave evaluates to be $1 + C\cos(\frac{2\pi x}{p'})$ where C is the contrast. And finer pitches are realized if θ is increased.

Let us now consider a symmetric configuration shown in Fig. 6, where only the zeroth and the minus first-order beams are admitted. [This simple configuration for demonstrating maximum achievable resolution of a projection imaging system was first shown to me by Hank Smith in his Submicron Structures class (MIT course 6.781) in the spring of 1986.] To simplify our analysis, we let the system have unit magnification. (The conclusion $p_{\min} = \frac{1}{2}\frac{\lambda}{\mathrm{NA}}$ remains valid if the system magnification is different from unity. In that case, we must distinguish the two NA's: the object-side NA and the image-side NA. Their ratio is the system magnification.) Due to symmetry and because $p' = \lambda/(2n \sin\theta)$, we have $\theta_i = \theta_0 = \theta$, $\theta_{-1} = -\theta$, and $p = p'$. The imaging system attains its maximum resolution when the two diffracted beams just reach the aperture of the imaging optics. This maximum resolution is then

$$p_{\min} = \frac{\lambda}{2n \sin\theta_{\max}} = \frac{1}{2}\frac{\lambda}{\mathrm{NA}}$$

as was shown by Abbe, since $\mathrm{NA} \equiv n \sin\theta_{\max}$.

Abbe's formulation has several advantages over Rayleigh's for us lithographers. First, working with periodic features rather than the more ambiguous two-point or two-line object, Abbe clarified for us that the $\frac{1}{2}\frac{\lambda}{\mathrm{NA}}$ resolution limit on pitch is not for convenience but is absolute, beyond which there is no resolution at all; he also pointed out how this resolution limit could be obtained in practice. Second, Abbe's explicit use of NA means the index of refraction is included, and hence, NA can be made greater than unity which Abbe and others had already put into practice in microscopy by then and which was also put into practice in microlithography early in this century. Finally, the resolution limit is always about pitch and not the linewidth of a feature. We lithographers know well that the linewidth of a feature can be made (theoretically infinitely) small by e.g., overexposing a positive-tone photoresist.

Lord Rayleigh was a master physicist (J.D. Jackson's words) who worked on diverse topics of physics and his name deserves our utmost admiration. In this case, however, Abbe was there first, and his investigations were more relevant to microlithography that we practice today. Hence perhaps the resolution formula in microlithography ought to more appropriately be named the Abbe formula rather than Rayleigh's equation.

Acknowledgments

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