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REVIEW

Trajectory generation and optimization for five-axis on-the-fly laser drilling: a state-of-the-art review

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Abstract. The process of on-the-fly laser drilling is capable of achieving high throughputs and offers a highly productive approach for producing predefined groups of holes (clusters) to be laser drilled on freeform surfaced parts (e.g., gas turbine combustion chamber panels), current machine tool controllers are not equipped with appropriate trajectory functions that can take full advantage of the achievable laser drilling speeds. While the problem of contour following has received previous attention in time-optimal trajectory generation literature, on-the-fly laser drilling presents different technological requirements, needing a different kind of trajectory optimization solution. This paper presents industrial state of the art and a literature review in the area of trajectory generation/planning and optimization for robots and in particular, machine tools, and laser drilling technology. [DOI: 10.1117/1.OE.57.12.120901]

Keywords: trajectory generation; trajectory optimization; five-axis; on the fly; laser drilling.

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1 General Overview of Laser Drilling

Manufacturers of turbine engines for aircraft propulsion and power generation have benefited from the productivity of lasers for drilling small (0.3- to 1-mm diameter) cylindrical holes at 10 deg to 90 deg to the surface in cast, sheet metal, and machined components. The ability to drill holes at shallow angles at high rates per second has enabled new designs incorporating film-cooling holes for improved fuel efficiency, reduced noise, and lower nitric acid (NO), nitrogen dioxide (NO_2), and CO emissions.[1]

Common techniques used in laser drilling are percussion hole drilling, on-the-fly drilling, and trepanning. Percussion drilling is a process where multiple pulses are applied per hole, while the part is stationary, to disperse sufficient material to open up the hole cavity.[2] The on-the-fly drilling process is where the holes are drilled with a single shot at a time, while the part is in relative motion with respect to the laser beam, and the shots are repeated as required to open up the holes.[3] Trepanning is a process where certain contours are cut by drilling closely spaced holes. Each of these laser drilling techniques will be explained further in this paper. Compared with percussion drilling, on-the-fly laser drilling offers important advantages, which are as follows:[4]

- Better material properties and feature quality, due to reduced thermal loading on the part;
- Smoother axis motion (as opposed to stop-and-go movements, as required in percussion drilling), which reduces vibrations induced onto the laser optics;
- Less downtime for optics realignment (which would be caused by vibrations);
- Higher productivity as motion "smoothness" can be translated into higher processing speed.

However, the feasibility of using percussion drilling might surpass that of on-the-fly drilling techniques in certain scenarios, especially when the laser pulsing period is much shorter than the servo drives' positioning time. It must be predetermined whether on-the-fly drilling is feasible for a particular application. Therefore, there is a need to study trajectory and hole sequence optimization methods for both percussion and on-the-fly laser drilling.

Incremental improvements in laser process and control technologies have led to substantial increases in the number of cooling holes used in turbine engines. Fundamental to these improvements and increased use of laser drilled holes is an understanding of the relationship between process parameters, hole quality, and drilling speed.[5] Laser drilling is a successful manufacturing solution for many industries due to its advantages over conventional drilling techniques. Advantages include noncontact processing, low heat input into the material, flexibility to drill a wide range of materials, accuracy, and consistency. Other benefits include drilling submicron holes, small holes with large aspect ratios, and drilling at angles.[2]

Lasers can also be focused to spot sizes as small as 1 to 20 microns[2,6,7] and even spot sizes in the submicron scale for some applications.[8] Coupling the high peak power with short pulse widths, a laser beam offers very good drilling capabilities in thin sheets. The optics configuration is designed to achieve the right spot size required for drilling various hole diameters. High-power lasers are also used for rock drilling applications,[2] drilling of flow filters and strainers, submicron drilling in flexography ceramic rolls, high-speed drilling of guide vanes, hole drilling of silicon, drilling diamonds for removing imperfections, and on-the-fly drilling of cooling holes. Laser systems are also used to manufacture microholes in fuel injection components, vertical probe cards, metered dose inhaler products, pinholes and slits for scientific instrumentation, inkjet printer nozzles, sensors and detectors, high-resolution circuitry, fuel cells, fiber-optic interconnects, and medical devices. UV and visible laser have been used in drilling small holes in ceramics, diamond, silicon, and other semiconductors, polymers, glass, and sapphire. Other shapes of holes are also possible, such as rectangular and other complex geometries.[9–11]

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This paper focuses on trajectory planning for laser drilling of cylindrical holes in turbine engine components. This process occurs through melting and vaporization (also referred to as "ablation") of the work-piece material through absorption of energy from a focused laser beam.

Manufacturers are applying results of process modeling and experimental methods to better understand and control the laser drilling process. The result is higher quality and more productive processes that in turn lead to better end products, such as more fuel-efficient and cleaner-burning aircraft and power-generating turbine engines. To take full advantage of the improvements achieved in the process, there is also an urgent need to design suitable motion control trajectories.

In computer numerical control (CNC) of machine tools, the toolpath geometry and feedrate (i.e., time-dependent progression along the toolpath) are typically planned as separate tasks. Computationally intensive tasks, such as toolpath parameterization, can be either handled in the computer aided manufacturing (CAM) system or in the preprocessing by the CNC executive kernel. Feedrate generation and feed optimization are coupled tasks within the trajectory generation module of the CNC controller. Reducing machining cycle time along curved toolpaths relies on the ability of the feed optimization algorithm to command the feed motion along a toolpath, so as to drive the machine tool and process within the physical limits while maximizing productivity.

Compared with traditional machining operations, where the toolpath has to follow a continuous contour, on-the-fly laser drilling poses significantly different technological requirements. This process requires the travel duration between consecutive hole locations, which corresponds to the laser firing period, to be kept constant (or as an integer multiple of the laser pulsing period), and minimized throughout the part program. Motion paths between the holes, however, are not restricted in shape and can be modulated to allow for the maximum possible reduction in the laser firing period. As the drilling is realized while the part is in relative motion with respect to the beam, hole elongation also needs to be considered and capped to avoid violating the part tolerances. Furthermore, the sequence in which the holes are drilled needs to be optimized to ensure motion efficiency and a shorter drilling cycle time. A machine tool's five-axis kinematic structure[12,13] and velocity, acceleration, and jerk limits also need to be taken into account. Some of these issues have been considered and incorporated in the candidate's previous work; namely in generating time-optimized trajectories for given hole sequences and seamless, as well as jerk- and time-optimal, connections between optimized cluster (a group of holes) trajectories.

This paper focuses primarily on background literature related to percussion and on-the-fly laser drilling.

1.1 *Types of Laser Drilling Techniques*

Laser drilling provides a highly productive method for producing holes on freeform surfaced parts, especially sheet metal. There have been detailed studies that characterize the process of laser drilling[14,15] and evaluate various machine configurations.[16] This section provides a brief review of laser drilling methods, as well as advantages and disadvantages.

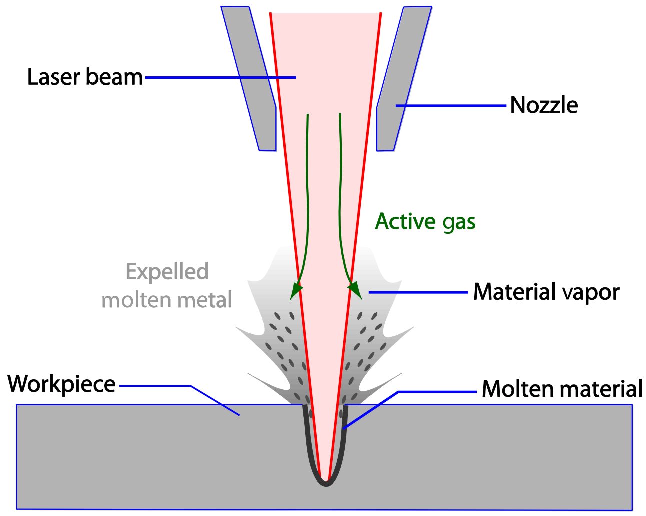

Fig. 1 How laser drilling works: the laser melts and vaporizes the material. The vapor pressure expels the molten material from the hole.

In laser drilling, a short laser pulse with high-power density feeds energy into the work-piece extremely quickly, causing the material to melt and vaporize. Figure 1 shows the basics of a laser pulse hitting the surface of a work-piece. The greater the pulse energy, the more material is melted and vaporized. Vaporization causes the material volume in the drilled hole to increase suddenly, creating high pressure. The pressure expels the molten material from the hole. Over the years, several drilling processes have developed from this basic method.

1.1.1 *Single-shot and percussion drilling*

In the simplest case, a single laser pulse with relatively high pulse energy can be used to produce a hole. This method enables a large number of holes to be created in an extremely short amount of time, compared with mechanical drilling methods. In percussion drilling, the hole is produced using multiple short-duration, low-energy pulses. This technique produces deeper, more precise holes than single-shot drilling, and also enables smaller hole diameters. Figures 2(a) and 2(b) show the difference between a single shot versus percussion drilling. On-the-fly laser drilling uses the single-shot laser drilling method while the machine axes are in continuous motion, by reducing the power of the shot and repeating the passes. This way, thermal energy build-up on the part can be greatly reduced. Thus, deep and narrow holes with excellent dimensional and material quality can be produced by applying on-the-fly drilling with repeated passes. However, the dynamic accuracy of the machine tool's motion is crucial for the successful application of on-the-fly drilling.

1.1.2 *Trepanning*

Trepanning uses multiple laser pulses to produce the hole. In this process, a pilot hole is first created using percussion drilling. Then the laser enlarges the pilot hole, moving over the work-piece in a series of increasingly larger circles.

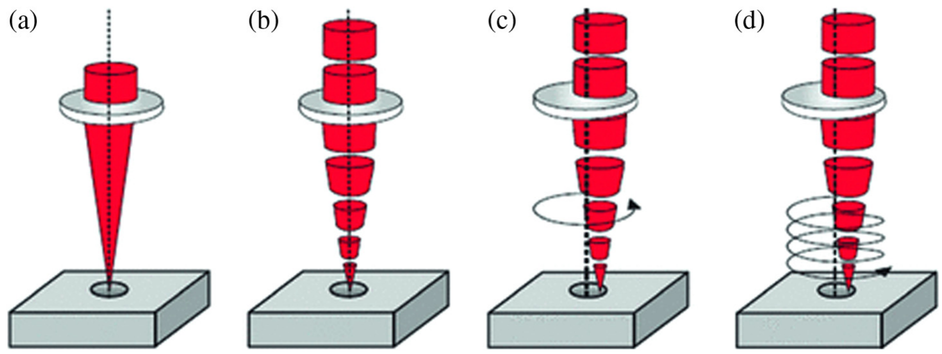

Fig. 2 Laser drilling techniques: (a) single pulse drilling, (b) percussion drilling, (c) trepanning, and (d) helical drilling (from left to right).[17]

Most of the molten material is expelled downward through the hole. Figure 2(c) shows this laser drilling method.

1.1.3 *Helical drilling*

Unlike trepanning, helical drilling does not involve the creation of a pilot hole. Right from the start, the laser begins moving in circles over the material as the pulses are delivered, with a large amount of material shooting upward in the process. The laser continues to work its way through the hole in a downward spiral. The beam focus point, meanwhile, can be adjusted so that it is always at the base of the hole. Once the laser has pierced the material, it can complete a few more revolutions to enlarge the base of the hole and smooth out the edges. Helical drilling makes it possible to produce very large and deep high-quality holes. However, there is persistent thermal loading at the hole bottom, which can have undesirable effects in terms of material properties. This operation is shown in Fig. 2(d).

The two main drilling techniques this paper focuses on, for trajectory planning, are percussion and on-the-fly laser drilling. These two methods are widely used in the aerospace industry in gas turbine engine production.

1.2 *General Advantages and Disadvantages of Laser Drilling*

Lasers can be used to drill holes in a variety of materials, ranging from wood and plastics to metals and ceramics. Typical examples of laser drilled holes in practical applications are cooling holes in aeroengine components, holes in fuel injection nozzles, ink-jet printer heads, and microvias in printed circuit boards (PCBs).[9,18]

Some of the main advantages of using lasers for drilling are:[18]

- Noncontact technique: The drilling medium is a beam of light; therefore, there is no physical contact between (moving) parts and the work-piece. This prevents contamination of the work-piece and (gradual) wear of the drilling part.
- High aspect ratios. With lasers, holes with aspect ratios (depth to width) of, for an example, 30:1 are easily produced. Furthermore, it has been shown that using nondiffractive beams, channels of high aspect ratios >100 can be produced in transparent materials, such as glass.[19]
- Holes at shallow angles: Laser drilling is particularly suited for drilling holes at an angle with the surface of the work-piece, for example, cooling holes in aeroengines. With laser drilling, holes at an angle as small as 10 deg with the surface can be produced.
- Drilling of difficult to process materials: Lasers can be used to drill a wide range of materials, from rubber and wood to very hard materials, such as diamond and ceramics.
- High speed and accuracy: Laser drilling is fast, accurate, and readily automated.
- Availability of photolytic drilling with photolytic processes: (i.e., those involving the breaking of chemical bonds for material removal, rather than melting and evaporation); virtually no recast layer and haze are formed, due to the fact that there is hardly any heat generation in some applications.

Disadvantages of the use of lasers for drilling may be:[18]

- High capital investment: The capital investment needed to purchase or custom develop a laser machine tool can be considerable.
- Thermal effects: Due to heating, a haze (surface thermal reaction and/or collection of expelled molten material—generally resulting in a cloudy and rough surface) may be present around the hole, particularly with pyrolytic processes (i.e., those involving heat generation). Furthermore, thermal shock may lead to microcracks in some materials.
- Hole edge and surface quality: With pyrolytic processes, due to the melting and evaporation of material, a recast layer and dross build up at the entrance and exit of the hole may be present. These reduce the repeatability and quality (for example fluid flow characteristics) of the holes.
- Taper in deep holes: Particularly in holes with a large aspect (depth to width) ratio, a considerable taper may be present, which may be unacceptable.

2 Five-Axis Laser Drilling Overview

Laser drilling provides a highly productive method for producing arrays of holes on planar- and freeform-shaped components. Industrial applications include fuel injection nozzles, PCBs, inkjet printer heads, pinholes and slits for

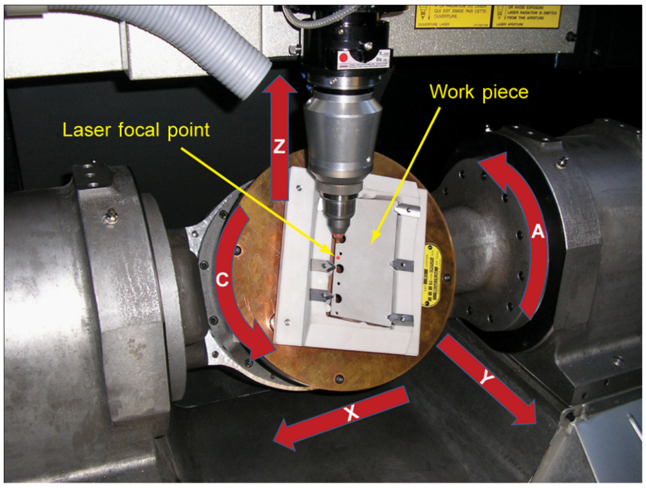

Fig. 3 Configuration of a five-axis laser drilling machine tool.

scientific instrumentation, high-resolution circuitry, sensors and detectors, fiber-optic interconnects, medical devices, and gas turbine combustion chamber panels.

Time-optimized trajectory generation has previously received attention in robotics and machine tool literature for contour following applications.[20–23] There have also been successful works for following way-point data by modulating the time intervals in between the points.[24] However, the nature of on-the-fly laser drilling requires the motion duration between consecutive holes, which corresponds to the laser firing period, to be kept constant and minimized. In between the holes, the motion path is not fixed and can be modulated to achieve the maximum possible time reduction. This presents a new type of trajectory optimization problem, specific to on-the-fly, and percussion laser drilling.

Figure 3 shows a five-axis laser drilling setup actuated by direct drive motors. Linear motors are used for motion in the *x*-, *y*-, and *z*-axes directions and the trunnion has a formation with two rotary axes (for rotary motions in the *a*- and *c*-axes). This machine was built for drilling gas turbine combustion chamber panel hole patterns like the one shown in Fig. 4. Figure 4 also shows the numbered collections or groups of holes (clusters) that need to be drilled, in this specific example; there are 12 different clusters to be drilled by means of an optimized smooth trajectory. It is obvious that on-the-fly drilling of such a pattern requires full coordination of all five axes. The hole clusters are determined in the Computer Aided Design and Manufacturing software and each cluster needs to be drilled at a fixed laser pulsing frequency. After drilling a single cluster, the connection between the clusters also has to be seamless with continuous smooth motion instead of decelerating and stopping at the end of one cluster, repositioning at the beginning of the next cluster and accelerating at the start of the drilling process for the consecutive clusters. The seamless cluster connection is performed to avoid unwanted vibrations on the machine and laser optics induced by aggressive and repetitive stopping and starting motions during the process. Hence, minimizing the duration of both cluster drilling and repositioning, while respecting the physical limitations of the machine and process, is key to achieving high productivity in this operation. Currently, there exists no commercial interpolator or published technique prior to this study, which generates time-optimized trajectories for on-the-fly laser drilling.

2.1 *On-the-Fly Laser Drilling versus Percussion Drilling*

As previously discussed, there are several methods of laser drilling. Related to five-axis laser drilling, one common method used is percussion drilling, where a series of laser pulses are sent to each hole while the component being drilled remains stationary. Each pulse causes a certain volume of material removal through ablation, and the laser pulses continue on until the hole is completely opened up. Then, drilling of the next hole proceeds by repositioning the part with respect to the beam. The percussion drilling method is highly productive, especially when the laser pulses can be delivered at high frequency. However, there is local thermal loading on the part, which may deteriorate the material properties and hole quality. Furthermore, obtaining the optimum sequence of positioning trajectories is an important and open research problem, especially for drilling configurations involving more than three simultaneous translational axes. Optimizing the beam positioning sequence and trajectory subject to the capabilities of a laser drilling machine and process enables minimum cycle time, therefore maximum productivity. Achieving this optimization for

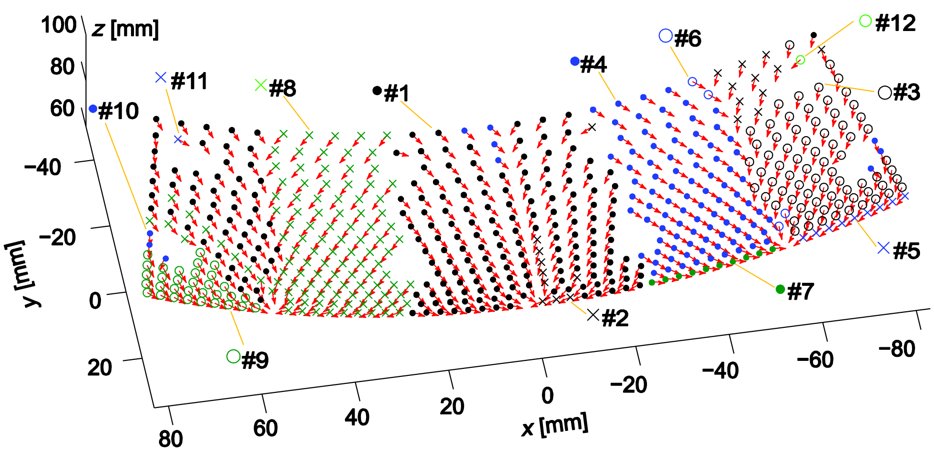

Fig. 4 Hole clusters and orientations for a turbine combustion chamber panel.

percussion laser drilling in an integrated manner, by jointly considering the sequencing and trajectory optimization problems for beam positioning in five-axis coordinates, is one of the discussed topics in this paper.

Another laser drilling method, which is advantageous in terms of resulting in better material properties and part accuracy, is on-the-fly drilling. In this method, each hole receives only one low-powered shot at a time while the work-piece is in continuous motion with respect to the beam. The positioning sequence repeats itself until all holes are gradually opened up in small increments. In this method, each drilled hole location has ample time to cool down before the next shot is received. Thus, on-the-fly drilling can result in more favorable material properties in terms of preserving the desired crystal structure around the hole, and better quality in terms of dimensional (size) and form (shape) accuracy. However in the case of on-the-fly drilling, the trajectory planning and sequencing become even more complicated tasks as there is no industrially available trajectory planner specifically designed for this operation (especially for five-axis movements). There is only a limited amount of literature that mainly targets two-axis sequencing for percussion type drilling operations. Hence, another discussion in this paper is the development of an integrated sequencing and optimized trajectory planning method for five-axis on-the-fly laser drilling.

2.2 *Feasibility of On-the-Fly Laser Drilling*

On-the-fly laser drilling may not always be the most productive solution, especially when precision drilling requires each hole to be drilled with multiple laser shots. In this case, percussion drilling (i.e., coming to a full stop at each hole and firing a sequence of shots) may be a more productive solution. In percussion drilling, the drilling frequency can also be increased to speed up the process. However, a drop in the laser power, due to higher pulsing frequency, can also be expected. In practice, this is compensated by firing more shots per hole.

The following analysis investigates the time efficiency of both methods and shows the condition for which on-the-fly drilling produces a shorter cycle time:

N: Number of holes in a single cluster,

n_{Fly}: Number of shots per hole for on-the-fly drilling,

n_{Per}: Number of shots per hole for percussion drilling,

T_{Fly}: Average segment travel duration for on-the-fly drilling,

T_{Per}: Average duration for hole repositioning in percussion drilling,

T_L: Laser firing period in percussion drilling (while axes are at rest),

D_{Fly}: Total process duration when on-the-fly drilling is used, and

D_{Per}: Total process duration when percussion drilling is used,

The total duration required for each operation can be expressed as

$$D_{\mathrm{Fly}} = N\, n_{\mathrm{Fly}} T_{\mathrm{Fly}}, \tag{1}$$

$$D_{\mathrm{Per}} = NT_{\mathrm{Per}} + Nn_{\mathrm{Per}} T_L. \tag{2}$$

For on-the-fly drilling to be more time efficient than percussion drilling, Eq. (3) must hold

$$D_{\mathrm{Fly}} < D_{\mathrm{Per}}. \tag{3}$$

Substituting Eqs. (1) and (2) into (3)

$$Nn_{\mathrm{Fly}} T_{\mathrm{Fly}} < NT_{\mathrm{per}} + Nn_{\mathrm{per}} T_L.$$

Resulting in

$$T_{\mathrm{Fly}} < \frac{T_{\mathrm{Per}}}{n_{\mathrm{Fly}}} + \frac{n_{\mathrm{Per}}}{n_{\mathrm{Fly}}} T_L. \tag{4}$$

Considering a simple example where $n_{\mathrm{Fly}} = n_{\mathrm{Per}}$ (i.e., no power drop due to higher frequency laser pulsing) and $T_{\mathrm{Fly}} = T_{\mathrm{Per}} = T_L$ (i.e., the machine tool's feed drives are fast enough to re-position the holes at the pulsing rate of the laser), it can be verified that

$$T_{\mathrm{Fly}} < \left(1 + \frac{1}{n_{\mathrm{Fly}}}\right) T_{\mathrm{Per}}. \tag{5}$$

For a case involving eight laser shots per hole, it can be verified that on-the-fly drilling will be at least 11% faster than percussion drilling. For two shots per hole, the speed increase becomes 33%.

However, in practical cases, the laser frequency is faster than the hole repositioning speed of the feed drives, which is the main motivation behind developing a time-optimized trajectory generation algorithm for on-the-fly laser drilling. Such an algorithm should ideally satisfy the condition in Eq. (4), which makes on-the-fly laser drilling more time-efficient than the alternative method of percussion drilling.

In addition to cycle time, the vibration delivered to the machine structure, particularly the laser optics, also plays a vital role in determining the productivity of a laser drilling operation. Excessive vibrations can cause the optics to lose alignment quickly, thereby requiring extensive downtime for realignment. Rather than stopping at each hole, as is the case in percussion drilling, the continuous motion employed by on-the-fly drilling can dramatically reduce the high-frequency content in the acceleration profiles, by reducing the jerkiness of the motion commands. This, in turn, can lead to a significant improvement in the overall productivity of the process. Hence, kinematic cycle time alone cannot be used as the sole deciding factor in choosing between on-the-fly and percussion drilling. The impact of the process parameters and trajectory used in each operation, on the overall productivity, cost-effectiveness, and part quality also needs to be considered.

3 Literature Review

In CNC of machine tools, the toolpath geometry and progression along the geometry (i.e., feedrate) are typically planned as separate tasks, similar to the schematic in Fig. 5.

Figure 5 shows that intensive tasks (computationally) such as the toolpath segment arc-length integration and parameterization are handled by the CAM system offline, on the other hand, feed generation and trajectory interpolation

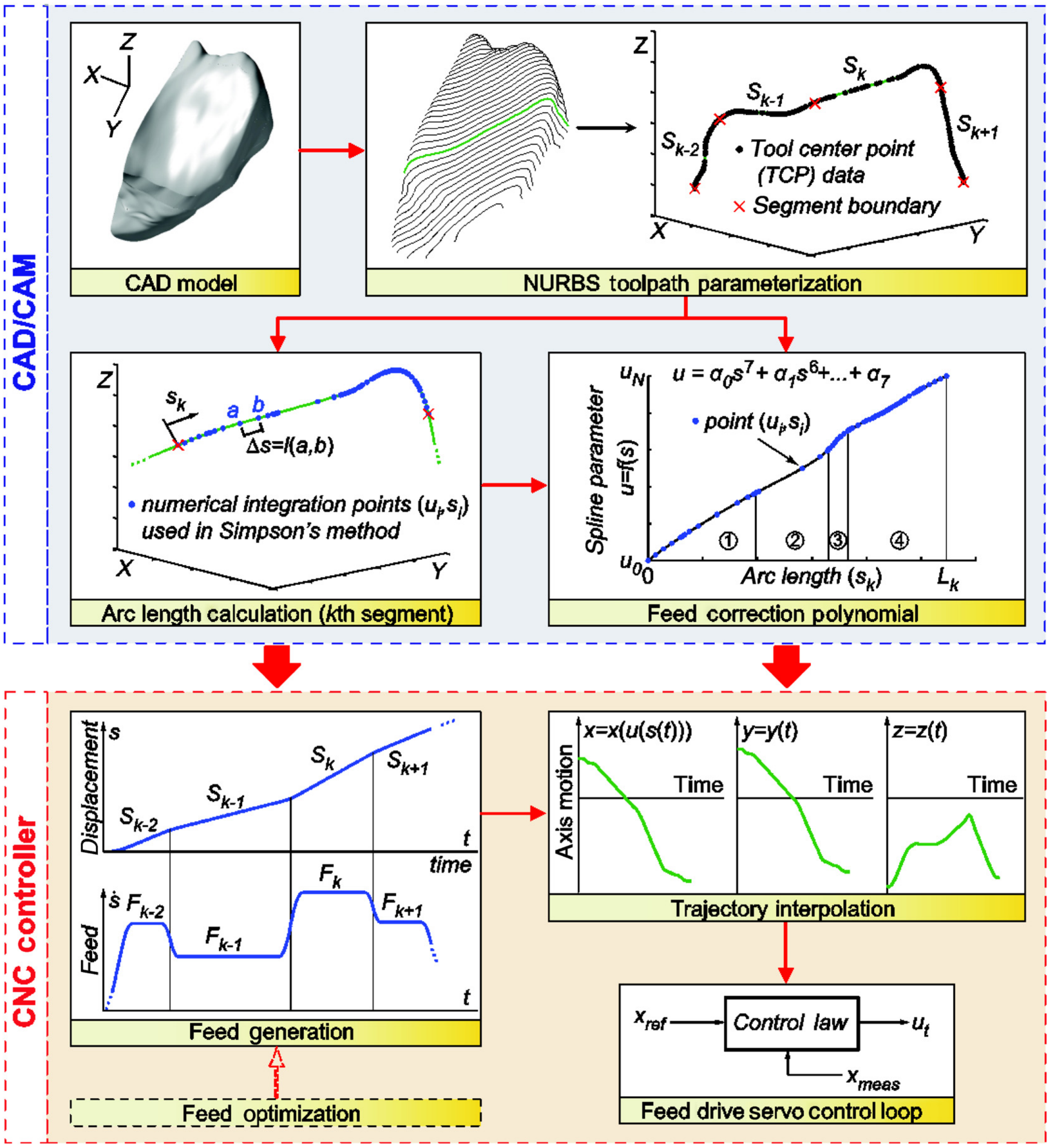

Fig. 5 Overview of trajectory generation in machine tools (From Heng[25]).

are realized in the CNC controller in real time. Feedrate generation and optimization are interfaced subtasks of the trajectory generation module in the CNC controller.

Nevertheless, they both influence the smoothness of the final interpolated trajectory. Considering that a point along a path defined in Cartesian space can be represented in vector form: $\boldsymbol{r} = \boldsymbol{r}(s) = [x(s)y(s)z(s)]^T$ as a function of the path parameter s, coming up with the definition of $\boldsymbol{r} = \boldsymbol{r}(s)$ constitutes the path planning task, and determining the progression along the path as a function of time [i.e., $s = s(t)$] is the feedrate planning task. The velocity, acceleration, and jerk profiles can be determined by applying the chain rule as follows:

$$\dot{\mathbf{r}} = \mathbf{r}_s\dot{s}$$
$$\ddot{\mathbf{r}} = \mathbf{r}_{ss}\dot{s}^2 + \mathbf{r}_s\ddot{s}$$
$$\dddot{\mathbf{r}} = \mathbf{r}_{sss}\dot{s}^3 + 3\mathbf{r}_{ss}\dot{s}\,\ddot{s} + \mathbf{r}_s\dddot{s}. \tag{6}$$

Above, $\mathbf{r}_s = d\mathbf{r}/ds$, $\mathbf{r}_{ss} = d^2\mathbf{r}/ds^2$, $\mathbf{r}_{sss} = d^3\mathbf{r}/ds^3$, $\dot{s} = ds/dt$, $\ddot{s} = d^2s/dt^2$, and $\dddot{s} = d^3s/dt^3$. It is clear that to get a smooth trajectory with continuous profiles up to acceleration level, and bounded profiles up to jerk level, the corresponding geometric ($\mathbf{r}_s$, $\mathbf{r}_{ss}$, and $\mathbf{r}_{sss}$) and time derivatives ($\dot{s}$, $\ddot{s}$, and $\dddot{s}$) also need to satisfy similar conditions for continuity and boundedness. This has motivated extensive research in trajectory generation methods in terms of toolpath planning, interpolation, and feedrate generation, as will be explained in Secs. 4 and 5.

When allowable, modulating the feedrate to achieve the shortest possible cycle time contributes to the productivity of the manufacturing operation being carried out. However, except for very simplistic cases, where only velocity and acceleration limits are considered, coming up with a time-optimal feed profile that limits the axis jerk values is a nontrivial task. The work conducted in this area has also been summarized in Sec. 5.

Compared with traditional machining operations, where the toolpath has to follow a continuous contour, on-the-fly laser drilling poses significantly different technological requirements. To the best of the author's knowledge, trajectory optimization for on-the-fly laser drilling has not received extensive investigation prior to this review or work done in Refs. 1, 3, 26, and 27. On-the-fly laser drilling requires the travel duration between consecutive hole locations, which corresponds to the laser firing period, to be kept constant and minimized throughout the part program. The toolpaths between the holes, however, are not restricted in shape and can be modulated to allow the maximum possible reduction in the laser firing period. As the drilling is realized while the part is in relative motion with respect to the beam, hole elongation needs to be considered and capped to avoid violating the part tolerances. The hole elongation constraint is explained in detail in Refs. 1 and 3. In addition, the machine tool's five-axis kinematics and velocity, acceleration, and jerk limits also need to be taken into account. These issues have been considered and incorporated into the trajectory optimization algorithm developed in Refs. 1, 3, and 26. A brief review of the existing work related to laser drilling is presented in Sec. 5. The paper ends with a summary in Sec. 7.

4 Toolpath Planning and Interpolation

It is well known that discontinuities in the position commands can lead to large spikes in the velocity, acceleration, and jerk profiles. This, in turn, results in undesirable high-frequency harmonics in the motor force or torque, which can excite the natural modes of the mechanical structure or servo control system. Figure 6 shows a comparison between velocity-, acceleration-, and jerk-continuous motion. As the motion becomes smoother, the high-frequency content in the acceleration harmonics diminishes dramatically, thus reducing the high-frequency excitation delivered to the machine tool structure.

High-frequency harmonics can also cause actuator saturations (by pushing the actuators beyond their functional limits) or axis tracking errors as a result of actuator saturation, meaning that the axes are incapable of following the reference position commands, thus causing deviations from the desired trajectory, thereby resulting in violations of the part manufacturing tolerances. Considering this effect, using only linear and circular interpolation techniques such as dies, molds, turbine blades, and aerospace parts (considered machine complex shapes) has serious limitations in term of productivity as the machine tool must decelerate/accelerate or stop between consecutive G codes.[28,29] Therefore, a great deal of work has been done to overcome these problems by developing spline toolpath definitions for three[25,30–42] and five-axis machine tools,[43–47] which yields paths with a second order or higher levels of continuity.

One of the main issues with spline toolpath planning is that the curve parameter (shown with u in Fig. 7) is not necessarily equal to the spline arc length (shown with s in Fig. 7). Since, in general $ds/du \neq 1$, the values of the spline parameter have to be carefully computed for each desired arc increment, to avoid inducing unwanted speed fluctuations. As measures to solve this problem, researchers have tried to either parameterize the spline toolpath to keep the value of ds/du as close to one as possible,[32–34] or they have devised Taylor series, feed correction polynomial-based, or iterative interpolation methods, which minimize unwanted feed fluctuations while interpolating the spline toolpaths.[25,29,31,34,39,48–55]

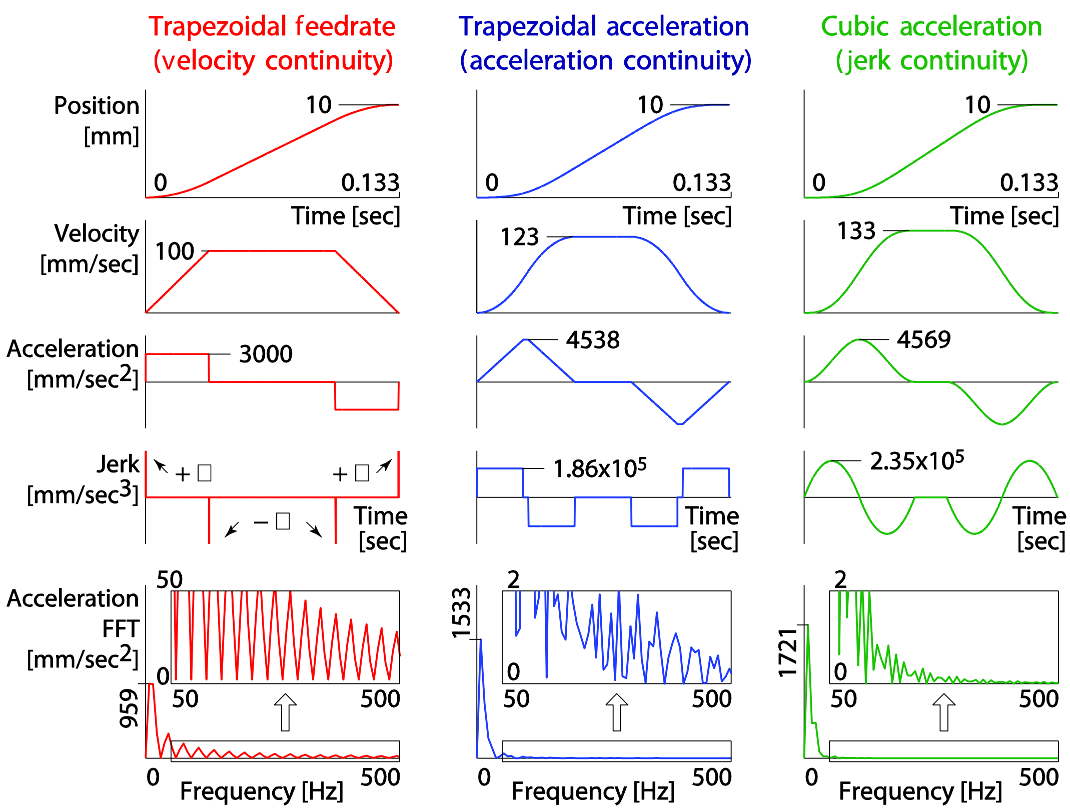

Fig. 6 Time- and frequency-domain comparison of three different trajectory types.

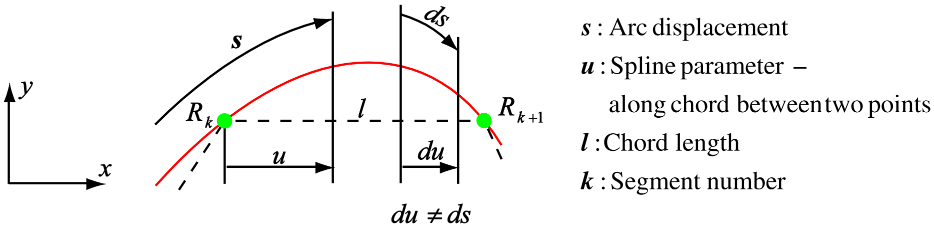

Fig. 7 Parameter (u) and actual path (s) increments in spline interpolation.

During on-the-fly laser drilling, as the toolpath is not fixed between the hole locations, maintaining constant feed is less of an issue, but coming up with an appropriate toolpath that will allow the highest travel speed while keeping the relevant kinematic profiles within the machine's and process' limits is crucial.

5 Optimal Trajectory Generation Methods and Optimization

5.1 *General Trajectory Optimization*

There has been a lot of research in the literature in generating optimized trajectories that pass through (or near) given waypoints. While some of the research has focused on generating the toolpath (geometry) and feedrate (progression speed) separately, other works have attempted to solve the commanded actuator trajectories directly as a function of time.

In trajectory generation, it is essential to have continuous acceleration profiles and bounded jerk, to avoid generating unwanted high-frequency content in the motion commands. Hence, different jerk bounded[31,56–58] and jerk continuous[39,59–63] motion planning techniques have been proposed in the literature. In addition, when the manufacturing process allows, optimizing the feed profile to minimize the cycle time can result in significant cost savings and productivity increase. Some of the feed optimization work has been pioneered in the robotics and machine tool literature with Refs. 20 and 64–66, which at initial stages resulted in acceleration discontinuous trajectories that were fast, but detrimental to production machinery. Later, as jerk and torque rate limits and the cutting process model were considered, various feed and trajectory optimization methods have emerged, which have been shown to be more effective.[21–24,39,65,67–74] Some of these methods make some kind of optimality trade-off in favor of faster computational speed, which is often in the form of constraining the feed profile to well-known shapes for an easy mathematical solution or adopting conservative feed limits based on the worst-case assessment. Elaborate techniques like the one in Ref. 21, which utilize full-blown sequential quadratic programming,[75] typically yield the shortest cycle times. However, such complicated methods are not always practical for reliable industrial implementation. Ideally, the solution sought in Refs. 1, 3, and 26 for on-the-fly laser drilling should be both easy and simple to implement, and also converge closely to a globally optimal solution (with minimal restriction on the trajectory profile shapes). Although offline implementation is targeted, excessive processing times are not acceptable.

There have also been studies to generate quick and smooth actuator trajectories by minimizing the integral square of jerk,[76–80] which has its roots in characterizing the movement of humans and primates.[81] This idea was first proposed for robotics and also applied in machine tool feed optimization for generating a parametric feed profile.[21] Furthermore, in Refs. 1 and 3, this idea has been taken one level further, by investigating the outcome of minimizing the integral square of the fourth time derivative (i.e., "snap"), which has been found to yield an initial guess that is very close to the desired time-optimal trajectory for on-the-fly laser drilling.

5.2 *Research on Trajectory Optimization for Five-Axis On-the-Fly Laser Drilling*

Laser drilling provides a highly productive method for producing hole clusters on freeform surfaced parts. Although there have been detailed studies that characterize the process of laser drilling[14,15] and evaluate various machine configurations,[16] only a limited amount of prior work has been done related to trajectory planning in this area.[82–84] To the best of the author's knowledge, trajectory optimization for on-the-fly drilling has not even been studied prior to Refs. 1, 3, 26, and 27.

Although the algorithm in Ref. 83 considers the optimal sequencing of hole locations based on the travel distance, the trajectory generation technique in Refs. 1 and 3 assumes that the holes are already sequenced by the computer aided design/manufacturing (CAD/CAM) software, and solves the time-optimal solution for traversing these holes on-the-fly. Reference 84 solves a general time-optimal trajectory problem in the presence of obstacles but does not take into account the process constraints related to on-the-fly laser drilling, such as the fixed laser pulsing frequency or the hole elongation problem. It deals with the problem of determining the optimum route for an end effector that visits a number of task points in a similar but not identical fashion to the well-known traveling salesman problem. The authors suggest that the measure be optimized is time instead of distance, and the travel time between two points is significantly affected by the manipulator configuration. Therefore, solutions of the inverse kinematics problem need to be taken into consideration[74] and it provides process models and trajectory planning techniques for preserving sharp cornered geometries during laser cutting.

References 1 and 3 present the time-optimized trajectory solution for the case where clusters of holes are presequenced, and they need to be drilled at a constant laser pulsing frequency. To ensure that hole elongation does not cause tolerance violations, the five-axis kinematics of the machine tool are also considered.[12,13,85] Axis-level velocity, acceleration, and jerk limits are considered throughout the part program. Rather than following the traditional method used in machine tool trajectory planning, by planning the toolpath and feed profile separately, the kinematic profile for each axis is directly formulated as a function of time. This greatly simplifies the solution of the optimization problem.

In Ref. 1, following the problem definition stated in Chapter 3, Cycle time optimized trajectories for each cluster are solved using the technique devised in Chapter 4. These trajectories are stitched together, or looped back onto themselves (for repeated laser shots), using the algorithm in Chapter 5. The intermediate and final results obtained have also been published in Refs. 3 and 4. Currently, the algorithm is being integrated for use in the production of gas turbine combustion chambers.[26,27]

During on-the-fly laser drilling, as the toolpath is not fixed between the hole locations, maintaining a certain feed profile is less of an issue; but coming up with an appropriate trajectory shape that will allow the shortest (quantized) travel durations between consecutive holes while keeping the relevant kinematic profiles within the machine and process limits is crucial. Due to the specialized nature of the operation, current CNC systems and CAD/CAM software do not offer support for on-the-fly laser drilling trajectory generation. Instead, customized solutions are co-developed by end-user companies and machine builders, based on the application.

References 1 and 3 had focused on developing time-optimized trajectory generation algorithms for traveling through given sequences of waypoints (i.e., hole locations). In Ref. 1, these sequences were determined using a customized algorithm, similar to the Nearest Neighbor approach explained in Sec. 2.4 of Ref. 1. Also, the sequencing algorithm developed at Pratt and Whitney Canada, and its principal details are given in Sec. 3.3 of Ref. 1 to provide context for the comparative simulation results presented in Chapter 3 of Ref. 1. Essentially, every next point is determined based on closest proximity using a weighted Euclidean two norms in five-axis coordinates, and each time the distance to the closest next point exceeds a given threshold a new cluster (a group of holes) of waypoints is initiated.

Based on these presequenced clusters, the following solution was developed (which is exemplarily shown in Fig. 8):

Step 1: **Trajectory "shape" optimization:** For each cluster of waypoints, the "shape" of the displacement profile is optimized as a function of time. This is done by modifying the first and second-time derivatives at the hole locations so that maximum time compression can be achieved for the displacement profile while holding all kinematic (velocity, acceleration, jerk, and hole elongation) constraints. This method makes use of the fact that when the timing of the displacement profile is scaled by a factor (α), the resulting velocity, acceleration, and jerk profiles are scaled inversely proportionally to the first, second, and third powers of α. During the shape optimization, it is assumed that every laser pulse would be used for drilling a hole within the cluster. Thus, the travel duration between the consecutive hole pairs is kept constant and equal throughout the sequence.

Step 2: **Time-optimized looping and stitching of cluster trajectories:** As on-the-fly drilled holes require repeated shots, the preoptimized clusters are connected back onto themselves as required, or time-optimal connections are made to consecutive clusters (when the necessary repeats are complete). The connection trajectories are planned to be integer multiples of the laser pulsing period in terms of duration, anticipating the use of the quick shutter in the optics path to divert unused pulses away during the positioning motion. These trajectories are designed to connect given boundary conditions of position, velocity, and acceleration while obeying velocity, acceleration, and jerk limits throughout the motion.

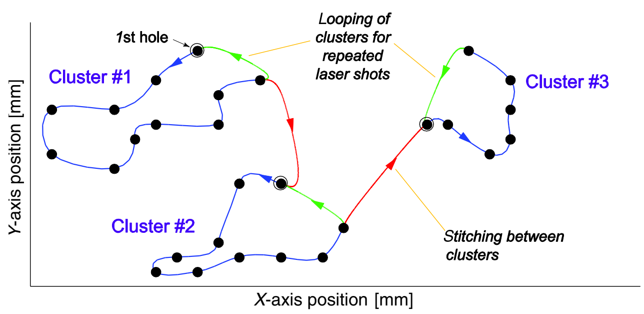

Fig. 8 Earlier developed strategy in Ref. 1 for on-the-fly drilling trajectory generation,[1,3] comprising of (1) optimizing each hole cluster separately and (2) looping and stitching of individual clusters using time-optimal connections.

In Ref. 1, following the idea of minimum-jerk splines, a "minimum-snap" quintic spline was used as the initial guess for fitting a spline through the given waypoints. This was followed by profile "shape optimization" to achieve the smallest time scaling factor, hence fastest trajectory for each cluster. This method was successfully applied in the trial production of several gas turbine components. The experimental results had demonstrated 13% to 46% reduction in the vibrations induced onto the laser optics, and also ∼10% improvement in the laser pulsing frequency (i.e., beam positioning time), compared with using the CNC system's existing trajectory planning function. The latter consists of blending linear toolpath segments using smooth corners at the waypoints. However, several issues were also identified with the developed algorithm, as listed in the following. Attempting the resolution of these issues has motivated and guided the majority of the research presented in Ref. 26.

- For each cluster, a minimum possible laser pulsing period is determined as a result of the displacement profile shape optimization. Upon the optimization of all clusters, the largest overall laser period is adopted for the whole trajectory. This is because, time-wise, it is very costly to alter the laser pulsing period on the fly. The power electronics in the current machine tool require a minimum of 40 s to discharge and recharge the capacitors, to mount a new laser "recipe." Thus, the most critical portion of a single sequence, which dictates a large laser period, ends up slowing down the process for the whole cluster, and thus the whole trajectory. Furthermore, when the sequencing of points is not done optimally, such critical regions emerge more often, which causes major bottlenecks in the productivity of the process. Hence, a profound need was identified to:

i. Improve the optimality of the sequencing algorithm in a way that considers the temporal nature of the commanded actuator trajectories.

ii. Enable travel durations between consecutive hole locations to be integer multiples of a laser pulsing period, rather than exactly one pulse, to enable further flexibility for slowing down during critical portions of a drilling trajectory, and also being able to go fast when the geometry allows.

- The second issue was that the "time-optimized" looping and stitching algorithm developed in the earlier work,[1,3] was based on an ad hoc approach without any optimality proof, or even proof of feasibility. In later benchmarks conducted, this method was seen to sometimes fail. In the sequencing and trajectory planning methods developed in Chapter 3 of Ref. 26, the need to use looping and stitching trajectories has been eliminated altogether, by removing the construction of clusters. However, there is still a need to perform time-optimal connections into and out of the repeating drilling trajectory, from and to rest (stopping) boundary conditions. In Ref. 26, these connections have been established with optimality proof and also feasibility analysis in Chapter 4, thus addressing the shortcomings of the earlier work in Ref. 1.

The combination of the new sequencing algorithm and time-optimal connection methods developed in Chapters 3 and 4 of Ref. 26 has resulted in 55% to 76% improvement in the motion cycle time over the earlier solution in Refs. 1 and 3 while staying within the same kinematic limits. The new trajectory planning approach has also been published in Ref. 27 and is being tested in further production trials at Pratt and Whitney Canada (jet engine manufacturer).

6 Possible Future Research

In the aerospace field, the discussed algorithms in this paper solve each cluster as a whole (by optimizing each cluster of holes as one complete set). Future work needs to focus on achieving the solution in moving windows so that trajectories for clusters with larger numbers of holes can be efficiently broken down into smaller subclusters and optimized without requiring excessive off-line computation time.

In addition, the hole sequencing currently applied in the CAD/CAM software was found to be one of the major bottlenecks that limited the achievable laser pulsing frequency. New sequencing techniques need to be investigated, similar to the Traveling Salesman approach, which will work concurrently with the trajectory optimization algorithms discussed in this paper to yield further cycle time reduction compared to what was achieved with the pre-set hole sequence.

Future optimization approaches also need to include finding the optimal hole clusters (a group of holes) to be drilled. This variable clustering of holes constrained by machine kinematics, coupled with finding the appropriate and optimized sequence of holes per cluster while considering other groups of holes, needs to be explored and is expected to provide further laser drilling cycle time reductions.

Furthermore, there is significant interest in the exploration of the theoretical globality of the solutions presented. This is a very significant academic challenge, therefore, methods such as interval analysis might be considered, which have been shown useful in finding global minima.

To further improve the field of trajectory generation and optimization, developing a virtual model of the machine tool dynamics, through multibody modeling, vibration modal analysis, and analyzing the feedback and feedforward control loops, would also enable the prediction of the servo errors for different drilling and positioning trajectories, without having to conduct time-consuming experiments on the actual machine tool. In this case, one easy correction would be to offset the position commands using means like iterative learning control in a virtual production environment, so that the actual beam positioning would be achieved on the actual part with the given tolerances. Such a model would also enable the prediction and containment of residual vibrations, especially in the orthogonal plane to the laser beam axis, which would further improve the part quality.

7 Summary

The process of on-the-fly laser drilling is capable of achieving high throughputs and offers a highly productive approach for producing pre-defined groups of holes to be laser drilled on freeform surfaced parts, current machine tool controllers are not equipped with appropriate trajectory functions that can take full advantage of the achievable laser drilling speeds. Although the problem of contour following has received previous attention in time-optimal trajectory generation literature, on-the-fly laser drilling presents different technological requirements, needing a different kind of trajectory optimization solution.

This paper presented a survey of industrial practice and academic literature covering multiaxis laser toolpath planning, feed generation for machine tools, and some of the issues specific to laser drilling have been presented in this paper. The challenges related to spline toolpath generation, interpolation, and feedrate optimization have also been discussed. Most recently, the proposed solutions related to on-the-fly five-axis laser drilling in Refs. 1, 3, and 26 differ from the traditional machine tool trajectory generation architecture and lends itself to an easier mathematical formulation and solution by formulating all of the kinematic profiles directly as a function of time.

The combination of the new sequencing algorithms and time-optimal connection methods developed in Ref. 26 has resulted in reducing the vibrations induced on the laser optics by up to 46% in some applications and 55% to 76% improvement in the motion cycle time over the earlier solution in Refs. 1 and 3 while staying within the same kinematic limits. The new trajectory planning approach has also been published in Ref. 27 and is being tested in further production trials at Pratt and Whitney Canada (jet engine manufacturer).

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